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Essays on Behavioural Economics and Household Finance

Tomasz Sulka

Doctor of Philosophy



The University of Edinburgh

School of Economics

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Declaration of Own Work

I declare that this thesis has been composed solely by myself and that it has not been submitted, in whole or in part, in any previous application for a degree. Except where stated otherwise by reference or acknowledgement, the work presented is entirely my own.

Tomasz Sulka

Dla Mamy i Taty.

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All errors and omissions are my own.

Abstract of Thesis

This thesis studies the impact of behavioural biases and limited cognition in the domain of household finance, in particular financial preparation for retirement. A special focus is placed on the resulting implications for policy-making. These questions are of particular importance in the current policy environment, which endows individuals with much responsibility for their economic decisions. Appropriate regulation and consumer protection appear to be an essential objective, and for two important reasons. First, the available financial products have become very complex and difficult to compare. Second, there is strong evidence for low levels of financial sophistication as well as for the impact of behavioural biases on financial decisions. Thus a helpful design of choice environments and regulatory policies requires a thorough understanding of the mechanisms underlying consumers' decision-making in a marketplace.

In Chapter 1, I modify the two-system model of self-control by introducing cognitive costs of decision-making in order to account for the difficulty of planning for retirement. The resulting possibility of (rational) inaction allows to generate a range of predictions supported by empirical evidence. Non-savers are characterised by poor financial self-control, high cognitive costs, and/or low incomes. Automatic enrolment into pension schemes has a substantial effect on participation (the 'default option effect'), but its impact on aggregate saving remains ambiguous as some individu-

als are ‘forced’ to save while others become ‘discouraged’ and adhere to potentially low default contribution rates. As a result, automatic enrolment reduces cross-agent variation in wealth accumulation. A stylised numerical application of the model produces substantial default effects and generates a U-shaped relationship between the default contribution rate and aggregate saving.

Savings behaviour may also be adversely affected by the presence of behavioural biases. In Chapter 2, I model a market for pension products in which a pension provider interacts with a present-biased individual. Agents who are aware of their future present bias (‘sophisticates’) are offered efficient savings contracts independent of the magnitude of the bias, while contracts offered to agents who are not fully aware of their future present bias (‘naifs’) are distorted in a way that exacerbates their forecasting errors. Importantly, such exploitative contracts can be either ‘inefficiently cheap’ (low-yield, low-fee) or ‘inefficiently expensive’ (high-yield, high-fee), depending on whether the income or the substitution effect of an interest rate change dominates in the agent’s utility function.

Chapter 3 constitutes a numerical extension of Chapter 2. To examine the quantitative importance of contractual design and choice, I introduce the interaction with a pension provider into a numerical life-cycle framework with hyperbolic discounting. A sizeable forecasting errors of a naïve agent affect the savings contract offered in market equilibrium in the predicted way. Under the benchmark calibration, the equilibrium contract is Pareto inefficient, lowers agent’s wealth at retirement by 10%, and generates a consumption-equivalent welfare loss of 0.17%.

Lay Summary

Ongoing policy changes make private pension arrangements increasingly important, relative to the traditional state-funded benefits (social security). As a result, individuals become more responsible for their financial security in retirement. However, complex financial decisions can be hindered by behavioural biases (systematic deviations from ‘rational’ decision-making) and limited cognition (limits to acquiring and processing information). In my thesis, I use economic theory to study how such imperfect decision-making translates into adverse outcomes. I subsequently explore a range of plausible policy interventions.

In Chapter 1, I introduce cognitive costs of decision-making into a model of financial self-control in order to account for the difficulty of planning for retirement. The resulting possibility of inaction allows to generate a range of predictions supported by empirical evidence. Non-savers are characterised by poor financial self-control, high cognitive costs, and/or low incomes. Automatic enrolment into pension schemes, whereby individuals need to take active action in order to opt-out rather than opt-in, has a substantial effect on participation (the so-called ‘default option effect’), but its impact on total saving remains ambiguous. That is because some individuals are ‘forced’ to save, while others become ‘discouraged’ and adhere to potentially low default contribution rates. As a result, automatic enrolment reduces variation in wealth accumulation across individuals. A stylised numerical application of the

model produces substantial default option effects and generates a U-shaped relationship between the default contribution rate and aggregate saving.

In Chapter 2, I model a market for pension products in which a financial provider interacts with a present-biased individual, i.e. an agent who is subject to temptation to overspend rather than save. Agents who are aware of their present bias (called ‘sophisticated’) are offered socially optimal savings contracts, while contracts offered to agents who are not fully aware of their present bias (‘naïve’) are distorted in a way that exacerbates their errors. Importantly, such exploitative contracts can be either ‘inefficiently cheap’ (low-yield, low-fee) or ‘inefficiently expensive’ (high-yield, high-fee), depending on the curvature of the agent’s utility function.

Chapter 3 constitutes a numerical extension of Chapter 2. To examine the quantitative importance of contractual design and choice, I introduce the interaction with a pension provider into a numerical life-cycle framework with present bias. A sizeable forecasting errors of a naïve agent affect the savings contract offered in market equilibrium in the predicted way. Under the benchmark calibration, the equilibrium contract is inefficient, lowers agent’s wealth at retirement by 10%, and generates a welfare loss corresponding to 0.17% of annual consumption.

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Chapter 1. Cognitive Costs and Corner Solutions in Retirement Planning

To account for the difficulty of planning for retirement, I introduce cognitive costs of decision making into the two-system model of self-control. The resulting possibility of rational inaction allows to generate a range of predictions supported by empirical evidence. Non-savers are characterised by poor financial self-control, high cognitive costs, and/or low incomes. Automatic enrolment into pension schemes has a substantial effect on participation (the ‘default option effect’), but its impact on aggregate saving remains ambiguous as some individuals are ‘forced’ to save while others become ‘discouraged’ and adhere to potentially low default contribution rates. Due to this mechanism, automatic enrolment reduces cross-agent variation in wealth accumulation. A stylised numerical application of the model produces substantial default effects and generates a U-shaped relationship between the default contribution rate and aggregate saving.

1.1 Introduction

Financial preparation for retirement appears to be a difficult task for an average individual. This is especially true given the ongoing pension reforms which reduce the generosity of state-funded benefits and increase the importance of private pension arrangements, predominantly of the defined-contribution type (Lusardi et al. 2017; OECD 2016, 2017). The underlying challenges can be separated into two distinct categories. First, an individual must be sufficiently financially literate in order to come up with an appropriate savings plan. Such a plan needs to indicate suitable savings vehicle, investment strategy, and desired contributions, for example. Second, an individual must exert enough financial self-control to be able to execute the plan. Both requirements are potentially problematic and therefore failures to save adequately emerged as an important policy issue (Benartzi, Thaler 2004, 2007). Importantly, these two stages of financial preparation for retirement are conceptually distinct. For instance, one can imagine that the individual's ability to follow through with a devised saving schedule should inform his decision whether or not to invest the effort into the necessary planning.

The intuitive notion that planning for retirement is challenging, but essential for wealth accumulation, is supported by the empirical evidence. In particular, Lusardi and Mitchell (2007), Binswanger and Carman (2012), and Van Rooij et al. (2012) document a large impact of planning on wealth holdings. In the sample of Lusardi and Mitchell (2007), for example, a median 'planner' has accumulated twice as much wealth as a median 'non-planner'. This positive impact of planning on wealth remains highly significant even after including a rich set of conventional controls. Moreover, the individual's planning behaviour is partially explained by his level of financial

knowledge.¹ A related strand of the literature studies the impact of various measures of cognitive ability on financial decision-making and saving outcomes (Banks et al. 2010; Banks, Oldfield 2007; Smith et al. 2010; Willis et al. 2014). One might bridge these findings by hypothesising that cognitive ability is simply an input into a decision to accumulate financial knowledge and plan for retirement, in the spirit of the model of costly financial knowledge accumulation by Lusardi et al. (2017). Simultaneously controlling for cognitive ability as well as for financial literacy provides a partial support for this mechanism (Lusardi, Mitchell 2014; Van Rooij et al. 2012; Willis et al. 2014).

Empirical evidence indicates also that the saving behaviour is significantly affected by self-control problems (Ameriks et al. 2007; Ashraf et al. 2006; Benartzi, Thaler 2004; John 2018). However, little is known about the interplay between planning for retirement and exerting financial self-control to follow the plan, in particular how these two inputs jointly determine the economic outcomes of interest.² In order to theoretically analyse the interplay between (costly) planning for retirement and self-control, in this chapter I modify the established two-system model of self-control (Benhabib, Bisin 2005; Brocas, Carillo 2008; Fudenberg, Levine 2006, 2012) by introducing the ‘cognitive costs of decision making’. The label ‘cognitive costs’ refers to any broadly defined psychological or mental costs associated with planning for retirement, such as investing in financial knowledge or processing information about the design of the pension system and the available savings vehicles. The idea to model difficult decisions as cognitively costly is not new, or particularly controversial (Kimball 2015).

¹See Lusardi and Mitchell (2014) for an overview of the literature that examines the role of financial literacy in determining economic outcomes.

²In a recent paper, Goda et al. (2018) show that exponential growth bias (a measure of financial illiteracy) and present bias (a measure of self-control) are uncorrelated and thus affect wealth accumulation through different channels.

In the model, an agent decides whether or not to bear the cognitive cost and plan for retirement, based on the expected utility gains from saving.³ As a direct result, the introduction of cognitive costs of decision-making allows for the possibility of corner solutions in the agent’s savings decisions or, in other words, inaction. This generates a range of predictions that are consistent with the observed saving behaviours. Specifically, the model implies that non-savers are characterised by poor financial self-control, high cognitive costs, and/or low incomes. I subsequently analyse the effects of introducing the automatic enrolment into pension schemes. The model predicts substantial impact on participation (the ‘default option effect’). That is because non-savers are likely to perceive automatic enrolment as welfare-improving and remain opted-in, even if the underlying pension scheme is ‘imperfect’ - actuarially unfair or inconsistent with their time preferences. However, the impact of automatic enrolment on aggregate saving remains ambiguous. While some individuals are ‘forced’ to save, others may become ‘discouraged’. The latter cease to save privately and rely on possibly low default contribution rates. Importantly, this implies that automatic enrolment into pension schemes reduces cross-agent variation in wealth accumulation. All three effects, i.e. the default option effect, ambiguous impact of automatic enrolment on aggregate saving, and equalisation of saving outcomes, have been robustly observed in the field (e.g. Benartzi and Thaler 2004, 2007; Choi et al. 2004; Madrian and Shea 2001). Nonetheless, I am not aware of alternative models that capture these effects simultaneously.

If the possibility of inaction is key to realistic predictions in the domain of retirement preparation, why not simply refer to an established model of procrastination by O’Donoghue and Rabin (1999a, 2001)? In the simplest terms, their model pro-

³Such cost-benefit approach seems natural, but poses some serious conceptual challenges, such as the infinite regress problem (Kimball 2015). Alaoui and Penta (2016) provide conditions under which the reasoning process can be represented by the cost-benefit analysis. They argue that these conditions are ‘weak and intuitive’, but also discuss the limitations of this approach.

duces procrastination due to the agent's naiveté about his self-control problem. At time t , the agent postpones an unpleasant (costly) activity, because he overestimates his willingness to carry out the task in the following period. However, when period $t + 1$ comes, the activity seems just as unpleasant and the agent postpones again. This mechanism is fundamentally different from the model of *rational* inaction proposed here. There is a conceptual distinction between procrastinating on a known best action (as in O'Donoghue and Rabin) and inaction due to the inability to infer the best option (as in the present chapter). Arguably, the latter mechanism is more suitable for thinking about retirement preparation, while the model based on naiveté about the self-control is more applicable to situations such as (not) going to the gym, unhealthy eating, or compulsive spending. In particular, Lusardi et al. (2017) discuss the emergence of 'rational financial ignorance' in their model of costly accumulation of financial knowledge. Importantly, the two models also yield different policy recommendations. For example, while forbidding delays in action-taking would improve the outcomes under the model of procrastination due to naiveté about one's self-control problem, it could be detrimental to welfare in the model of rational inaction. Finally, separating the cognitive costs from the self-control problems allows to study the interplay between the two channels in determining economic outcomes.

Some anecdotal evidence for rational inaction comes from the UK's 'Attitudes to Pensions' survey, which is conducted by the Department for Work and Pensions and used for policy advice. According to the 2012 edition of the survey, 19% of respondents had no private savings of any kind. Having no accumulated wealth was found to be positively correlated with reporting difficulties with financial decisions. As many as 71% of women (56% of men) agreed with the statement that 'sometimes pensions seem so complicated that I cannot really understand the best thing to do,' while 28% (13%) said that 'dealing with pensions scares them.' These findings are

paired with poor knowledge of the state pension system (MacLeod et al. 2012).

I subsequently present a stylised numerical application of the model in order to provide some additional intuition. The model produces large default option effects under reasonable parameter values. Moreover, the relationship between default contribution rates and aggregate saving is U-shaped, which highlights that automatic enrolment increases saving only when it is paired with high enough contribution rates. For automatic enrolment with default contribution rates proportional to the individual's earnings, the rate corresponding to a prospective minimum requirement under the UK law turns out to be welfare-maximising.

Based on the intuitive ideas underpinning the theory of mental accounting, I model self-control as exerted via a particular internal commitment device - 'savings targets' (Heath and Soll 1996; Levin 1998; Shefrin, Thaler 1988; Thaler 1985, 1990, 1999). These savings targets are meant to capture financial goals that an economic agent sets for himself in order to control his spending. From a perspective of that literature, the contribution of this chapter is to provide simple micro-foundations for the two building blocks of the mental accounting theory, namely non-fungibility of wealth and existence of category-specific financial goals. Discretionary saving and public pension wealth are *ex ante* only imperfect substitutes, but would be perfectly fungible in absence of cognitive costs and self-control problems. The model incorporates an endogenous decision whether or not to set a savings target, and what the target should be. Regarding the two-system literature (Benhabib, Bisin 2005; Brocas, Carillo 2008; Fudenberg, Levine 2006, 2012), this chapter introduces cognitive costs of decision making and the resulting possibility of inaction. It also constitutes a particular application of the two-system model of self-control to the domain of retirement-related decisions. Assuming that self-control is exerted via the savings

targets is only a framing modification.⁴

The remainder of this chapter is structured as follows. Section 2 outlines a two-period version of the model, derives its behavioural predictions, and briefly discusses an extension to a multi-period version. Section 3 focuses on the model's implications regarding automatic enrolment into pension schemes. Section 4 presents a stylised numerical application. Section 5 concludes. Derivations are relegated to the appendix.

1.2 The model

1.2.1 Economic environment

There are two periods indexed by $t \in \{1, 2\}$. The first period corresponds to a working life of an agent, while the second period illustrates retirement. Economic agent's disposable income is deterministic and equal to w (wage) and b (pension benefits) in periods 1 and 2, respectively.⁵ Income is the only source of wealth and it is further assumed that $w \gg b$. In order to simplify the analysis and focus attention on saving decisions motivated solely by an intention to smooth one's consumption over the life cycle, and not by precautionary motives, there is no uncertainty in the model. Accumulated savings earn real interest rate r and the model abstracts from the possibility of borrowing against one's pension income. There are no labour supply decisions.

⁴Arguably, the two-system model is more consistent with the findings from neuroscience than the popular alternatives - the model of present bias (Laibson 1997) and the temptation model (Gul, Pesendorfer 2001). The two-system model also does away with the issue of time-inconsistent preferences.

⁵Pension benefits are assumed to be funded with the agent's social security contributions, which alongside his gross wage determine w .

1.2.2 Preferences

The model assumes a dual structure of preferences, as in Benhabib, Bisin (2005), Brocas, Carillo (2008), and Fudenberg, Levine (2006, 2012). In period 1, a doer (the economic agent's impatient, myopic self) would like to maximise his instantaneous utility, while a planner (the economic agent's far-sighted, smoothing-oriented self) has an objective of maximising the lifetime utility, which is a discounted sum of instantaneous utilities in periods 1 and 2.

A doer has complete control over the available resources. However, a planner can exert some influence on a doer by using an internal commitment device in a form of a savings target. A savings target α is chosen exclusively by the planner, i.e. the doer treats the target as exogenously given. The actual level of savings chosen by the doer is denoted by a . Note that the intuitive ideas underpinning the theory of mental accounting are incorporated into the model by assuming the existence of (mental) savings targets - financial goals that are set by an economic agent for himself in order to control his spending.

The instantaneous utility in period 1 takes the following form:

$$U_1 = \log(C_1) - \Psi(a, \alpha),$$

where:

$$\Psi(a, \alpha) = \begin{cases} \psi(\alpha - a)^2 & \text{for } a \leq \alpha \\ 0 & \text{for } a > \alpha \end{cases}$$

where $\psi > 0$ and C_1 denotes current consumption. For simplicity, the instantaneous utility of consumption has a logarithmic form.

The savings target serves as an internal commitment device because a doer 'feels bad' (i.e. obtains negative utility) if he misses the target. This can be interpreted

as guilt that one feels when one fails to follow well-intended plans. Suppose that the intensity of this negative sensation is increasing in the amount by which the target was missed. The self-control parameter ψ determines the intensity of the sensation for a given (a, α) pair.

Nonetheless, it must be stated clearly that the above functional form is assumed purely for analytical convenience and therefore illustrates the trade-off between utility from current consumption and disutility from missing a target only in qualitative terms.⁶

The doer maximises U_1 subject to the budget constraint $C_1 \leq w - a$. The planner's aim, on the other hand, is to maximise the lifetime utility. However, setting the savings target is (cognitively) costly. Denote the cognitive cost of selecting the savings target by Φ . The planner's objective function is:

$$U = U_1 + \delta U_2 - \Phi(\alpha)$$

where

$$\Phi(\alpha) = \begin{cases} \phi & \text{if } \alpha > 0 \\ 0 & \text{if } \alpha = 0 \end{cases}$$

and $\phi > 0$ denotes the one-off cognitive cost of decision-making that is borne when the savings target is set. It is therefore implicitly assumed that the cognitive cost arises before one starts saving for retirement. This is consistent with interpreting ϕ as either effort of financial planning or the cognitive cost of acquiring relevant knowledge. The cognitive cost does not depend on the particular savings target - the effort required to come up with an appropriate saving schedule is independent of the

⁶In particular, the above function Ψ may enter the optimisation problem of the doer in a case-independent form without affecting the solution.

resulting plan itself. Potential real-life determinants of the level of cognitive costs may include baseline financial education, clarity of the pension system design, or stress-related cognitive load. Parameter $\delta \in (0, 1]$ denotes the intertemporal discount factor. In what follows, the lifetime utility function U is interpreted as a measure of the agent's welfare. This approach is consistent with the standard stand taken by the literature (for a related discussion, see Spiegler 2014).

The description of the model is completed by outlining the timing. First, the planner chooses the savings target. Subsequently, disposable income w becomes available to the doer and he decides how much to spend and how much to save given the target.

1.2.3 Characterisation of behaviour

Decision-making of the doer

The doer selects the actual level of savings a so as to maximise U_1 subject to the budget constraint $C_1 \leq w - a$. This yields the following unique solution:

$$a^* = \max \left\{ \frac{w + \alpha - \sqrt{(w - \alpha)^2 + \frac{2}{\psi}}}{2}, 0 \right\}$$

The derivation can be found in Appendix A. In case there is no savings target ($\alpha = 0$), the borrowing constraint binds and the doer does not save ($a^* = 0$). For a positive savings target, on the other hand, the larger ψ , which measures the magnitude of disutility for deviating from the target, the closer the actual savings made by the doer to the selected target. As ψ increases, a^* converges to α . It is thus clear why ψ can be interpreted as a self-control parameter. Economic agents characterised by greater ψ are able to exert more self-control in the sense that their actual saving levels are closer to the selected targets. Furthermore, even though the negative

sensation resulting from missing the savings target is increasing in ψ for a fixed deviation from the target, it is no longer the case once the actual saving behaviour of the doer a^* is taken into account. Because the actual saving is converging to the savings target as the self-control parameter increases, the total negative sensation is in fact decreasing in ψ for a given target α . Using terminology from the two-system literature, parameter ψ is inversely related to the cost of exerting self-control, i.e. the higher ψ , the less costly it is to control the doer's behaviour and induce a given level of saving.

There is no optimisation problem to solve in period 2. A retired doer simply consumes whatever wealth is available and obtains the utility of:

$$U_2 = \log(b + (1 + r)a)$$

Decision-making of the planner

Given the doer's decision rule, a planner has two options to choose from. He can either set a specific savings target for period 1, or set no target and allow the doer to consume his entire income. Which action leads to a higher level of lifetime utility depends on how large the benefits from consumption smoothing are relative to the cognitive costs. First, consider how levels of lifetime utility obtained at the optimum change with parameters ψ and ϕ at the optimum. Derivations of Lemmas 1 - 3 are relegated to Appendix B.

Lemma 1 (impact of self-control and cognitive costs on utility) *At the optimum, the lifetime utility obtained by an agent is:*

1. *weakly increasing in ψ ,*

2. *weakly decreasing in ϕ .*

The above result formalises the statement that economic agents characterised by higher ψ are able to smooth their consumption to a greater extent without bearing overly high costs of self-control. That is because the total negative sensation related to inducing a given level of saving is decreasing in ψ . The impact of cognitive costs on welfare is straightforward. They either directly decrease the lifetime utility of individuals who save, or prevent the remaining agents from smoothing their consumption.

Lemma 2 (possibility of inaction) *There exists non-negative and increasing $A(\psi)$ such that an agent does not engage in saving when $\phi \geq A(\psi)$.*

The above observation states that an individual will not save for retirement if the cognitive costs of planning are higher than the utility gains from consumption smoothing, represented by $A(\psi)$. As implied by Lemma 1, those gains are lower for individuals with low degrees of self-control and therefore the agents that are most likely not to save are the agents characterised by either high ϕ or low ψ . In other words, for any given level of cognitive costs, the poorer the self-control, the less likely it is that an agent will smooth his consumption over the life cycle. Furthermore, with high cognitive costs, small self-control problems might suffice to prevent an economic agent from saving. Cognitive costs and poor self-control can thus be seen as substitutes when considered as factors that prevent an individual from saving for retirement. One can also show that the utility gains from consumption smoothing are lower for economic agents with low incomes. To be more specific, for any

$\epsilon > 1$ and for any ψ , $A(\psi)$ is greater for an agent with an income profile $(\epsilon w, \epsilon b)$ than (w, b) .⁷

The ‘Attitudes to Pensions’ survey classified 19% of respondents as non-savers. Compared to the rest of the sample, these individuals were more likely to come from a low-income household and to exhibit symptoms of poor financial self-control and high cognitive costs of decision-making.⁸ These results are consistent with the prediction of the model. Lemma 2 highlights that the condition for not saving is more likely to be satisfied for low values of ψ , high values of ϕ , and a low income profile. Importantly, the resulting possibility of *rational* inaction is linked to poor financial self-control combined with the awareness thereof. This is in contrast to the established model of procrastination, which relies on naivete about one’s self-control (O’Donoghue and Rabin 1999a, 1999b, 2001).

The economic agent whose behaviour is described by standard theories is nested within the model as an agent with zero cognitive costs ($\phi = 0$) and perfect self-control ($\psi \rightarrow +\infty$). Call this individual the ‘classical’ agent. In case of the classical agent, the doer is completely obedient (i.e. $a = \alpha$) and therefore the savings targets devised by the planner are implemented without any disutility from missing the targets. The next lemma states how the optimal choices of an agent with self-control issues or non-zero cognitive costs compare with the choices of the classical agent.

⁷Although no assumptions are made regarding the correlations between parameters ψ , ϕ and income, one could easily imagine these characteristics as being interdependent. That is due to several empirical regularities. Some researchers suggest that cognitive abilities are positively correlated with self-control (e.g. Benjamin et al. 2013; Camerer 2013; Gathergood 2012). Moreover, individuals from low-income households experience more stress-related cognitive load (Mullainathan and Shafir 2013). Finally, incomes of highly-educated, and presumably characterised by higher cognitive abilities, tend to be high (e.g. Borjas 2010; Cahuc and Zylberberg 2004).

⁸More specifically, 29% of those individuals (compared to 59% in the rest of the sample) said that they were keeping up with their bills and credit commitments without any difficulties; 35% (58%) reported that they were putting some money aside for emergency situations; 41% (12%) said that ‘they would have no idea about what they needed to do when making important financial decisions, such as taking out a mortgage, loan, or pension’; 37% (16 %) reported their financial knowledge to be poor; 84% (51 %) stated that they had no idea what their retirement income would be (MacLeod et al. 2012).

Lemma 3 (upper bound for optimal saving) *Let a^C denote the optimal level of savings, as well as the savings target, of the classical agent. For the optimal choice of an otherwise identical agent with $\psi < +\infty$ or $\phi > 0$, it must be the case that $a^*(\alpha^*) \leq a^C$. Moreover, $a^*(\alpha^*)$ is increasing in ψ .*

In the above, $a^*(\alpha^*)$ denotes the level of savings optimal induced by the planner. Lemma 3 states that the savings made by any agent cannot exceed the savings of the classical agent at the optimum. In other words, a^C constitutes an upper bound for optimal savings, because agents with self-control problems cannot achieve the same extent of consumption smoothing as the classical agent without bearing an excessive cost of self-control. The planner considers the trade-off between consumption smoothing and disutility for missing high savings targets and therefore she finds it optimal to induce lower savings. Similar remarks apply when one compares two agents with imperfect self-control - an individual characterised by higher ψ optimally saves more. Furthermore, high cognitive costs may prevent an agent from saving altogether. As a result, the optimal saving $a^*(\alpha^*)$ is (weakly) decreasing in both self-control problems and cognitive costs.

The presence of cognitive costs and self-control problems results in public pension wealth and private savings being only imperfect substitutes *ex ante*. The following proposition formalises the notion of non-fungibility of these two kinds of wealth. What is important, in the setting of the current model, the result of non-fungibility does not rely on differences in liquidity across the two kinds of wealth.

Proposition 1 (non-fungibility of discretionary saving and public pension wealth) *If an agent is characterised by either positive cognitive costs or imperfect*

self-control, private saving and public pension wealth are not perfect substitutes, i.e. $a^(\alpha^*)$ is not linear in b . More precisely:*

1. *For $\phi = 0$ and $\psi \rightarrow +\infty$, $a^*(\alpha^*)$ is linear in b .*
2. *For $\phi > 0$ and $\psi \rightarrow +\infty$, $a^*(\alpha^*)$ is discontinuous, but piecewise linear in b .*
3. *For $\phi = 0$ and $\psi < +\infty$, $a^*(\alpha^*)$ is continuous, but non-linear in b .*
4. *For $\phi > 0$ and $\psi < +\infty$, $a^*(\alpha^*)$ is discontinuous and non-linear in b .*

See Appendix C for derivation. Under positive cognitive costs, the optimal level of private saving is changing discontinuously with b . As the public pension benefits become more generous, the utility gains from consumption smoothing decrease. Eventually, above a certain threshold for b , the utility gains become smaller than the cognitive costs of decision-making, preventing an economic agent from saving privately. At this threshold $a^*(\alpha^*)$ is discontinuous in b . The introduction of self-control problems results in the optimal level of saving being non-linear in the pension benefits (up to the threshold). That is due to the fact that when inducing the optimal $a^*(\alpha^*)$, the planner takes into account the negative sensation from setting the savings target, and not only the consumption-smoothing aspect of saving. Note that both self-control issues and cognitive costs would imply non-fungibility in isolation, but, as in case of Lemma 3, the impact of ψ and ϕ is exerted via separate channels.

1.2.4 Relation to the theory of mental accounting

Modelling internal commitment in a particular form of savings targets is motivated by the intuitive ideas behind the theory of mental accounting, according to which indi-

viduals simplify their spending-saving decisions by devising category-specific budget constraints.⁹ What is important, the current model provides some simple micro-foundations for the two building blocks of the mental accounting theory. First, the savings targets arise (or do not arise) endogenously. And second, the model generates non-fungibility of different kinds of wealth of the same liquidity.

1.2.5 Extension to a multi-period model

Extending the model to a version with $T > 2$ periods, where the last period corresponds to exogenously timed retirement, would be straightforward because the planner's preferences are time-consistent. The main results regarding the possibility of inaction, upper bound for optimal savings, and non-fungibility would be preserved by such extension. For example, Section 4 presents a stylised numerical application that employs a three-period version of the model.

A multi-period version of the model could be further extended in future work to provide some additional insight. As there would be an issue of potential early depletion of retirement wealth, one may consider the impact of liquidity of accumulated savings on the decision-making. If withdrawals were optimal in some periods, or in certain states of the world, this would give rise to the trade-off between commitment and flexibility (Amador et al. 2006; Galperti 2015). In such a setting, the interplay between internal and external commitment devices could also be analysed. Furthermore, assuming that cognitive costs evolve over time would allow to study the decisions about the optimal timing of retirement saving.

⁹Violations of these constraints carry penalties that are purely psychological.

1.3 Automatic enrolment into pension schemes

Automatic enrolment into pension schemes is introduced when undersaving for retirement emerges as a policy issue (OECD 2016, 2017). The richest empirical evidence on the impact of automatic enrolment on participation, contribution rates, and asset allocation comes from firm-level implementations in the US (e.g. Benartzi, Thaler 2004, 2007; Choi et al. 2004; Madrian, Shea 2001). Across studies, several regularities are robustly observed. First, automatic enrolment has a substantial positive impact on participation rates. Second, many of those automatically enrolled tend to passively accept default contribution rates and asset allocations. As a result, the impact of default coverage on aggregate saving is ambiguous. For instance, automatic enrolment paired with a low default contribution rate might have a negative impact on wealth accumulation for an average worker. Third, because individuals refrain from making active choices, automatic enrolment reduces variation in wealth accumulation across individuals. This section discusses the implications of introducing automatic enrolment using the model that accounts for cognitive costs of decision-making and self-control problems. The model can generate all the observed regularities, thus highlighting the importance of accounting for corner solutions (rational inaction) in individual retirement-related decision-making.

To consider a particular instance of an automatic enrolment reform as an example, refer to the introduction of automatic enrolment into workplace pension schemes in the UK by the Pensions Act 2008. As stated in the official report by the Department for Work and Pensions, “automatic enrolment is designed to harness the natural tendency towards inertia that people display in pension behaviour, by making people opt out, rather than opt in to a workplace pension” (MacLeod et al. 2012). Impor-

tantly, the reform also specifies minimum contribution rates and eligibility criteria.¹⁰ Among other, in order to be eligible for automatic enrolment, a worker cannot be a member of another qualifying pension scheme. It can thus be said that the automatic enrolment reform explicitly focuses on current non-savers. Moreover, even though employees can opt out of their workplace pension schemes, they must be automatically enrolled again every 3 years.

In case of the UK reform, during the first three years after implementation 5.5 million workers have been automatically enrolled into workplace pension schemes. The total number of participants increased by over 3 million, with the highest impact observed for low-earning and young workers. Opt-out rates were low at around 10%. Although the total amount saved into workplace pensions has been steadily increasing, the annual contribution per eligible saver fell by almost 27% (Department for Work and Pensions 2015). These observations match the regularities reported for the US.

The following discussion focuses on welfare and saving implications of the introduction of automatic enrolment. It must be noted that pension schemes are introduced into the model solely as a policy device which helps individuals smooth their consumption across the life cycle by collecting contributions in period 1 and providing pension benefits in period 2. In particular, any redistributive properties of the schemes are disregarded in the analysis. That is partly because most employers enrol their workers into individual-account, defined-contribution schemes (Clarke, Norris 2016).

Let $\hat{C}_1 = w - a^C$ and $\hat{C}_2 = b + (1 + r)a^C$ denote the optimal consumption path

¹⁰Automatic enrolment duties have been introduced sequentially between 2012 and 2018 based on employer size. Those employees who are at least 22 years old, but below State Pension Age, earn over £10,000 per year, and are not already members of a qualifying pension scheme are eligible for automatic enrolment. Currently, the total minimum contribution rate is equal to 5% of worker's earning, out of which at least 2% must be the employer's contribution. These rates will be raised to 8% (3%) in 2019.

of the classical agent. Note that any agent with the same income would maximise his lifetime utility if he could commit costlessly to consuming \hat{C}_1 and \hat{C}_2 , irrespective of the level of self-control or cognitive costs. The following, in contrast, defines an imperfect pension scheme.

Definition 1 (imperfect pension scheme) *Consider a pension scheme that collects a contribution of $\tau = (1 - z)w + za^C$ in period 1 and provides an additional benefit of $\beta = z(1 + r)a^C - (1 - z)b$ in period 2, for some $z \in (0, 1]$. Such a scheme effectively allows an agent to consume $z\hat{C}_1$ in period 1 and $z\hat{C}_2$ in period 2.*

The above definition of an imperfect pension scheme significantly simplifies the analysis, while allowing for useful interpretation. First, note that for $z = 1$, the scheme enables an agent to maximise the lifetime utility without bearing any self-control or cognitive costs. The scheme is not only welfare-maximising, but it also maximises aggregate saving and equalises outcomes across the agent-types differing in their ability to exert self-control and their cognitive costs. For a more interesting case of $z < 1$, a scheme can be interpreted as being actuarially unfair. That is because $\beta < (1 + r)\tau$, which means that the future value of provided benefits is strictly less than the future value of collected contributions. When interpreted in this way, parameter z can be said to capture any administrative costs of operating a pension scheme which lead to a discrepancy between the value of contributions and the value of associated benefits. However, the above definition allows for a broader interpretation. Note that by changing z continuously, one can replicate all levels of lifetime utility that are attainable to an agent with a given income profile. Thus z may be interpreted as illustrating any imperfections of a pension scheme into which an individual is automatically enrolled, from low investment returns to distributing

income across the life cycle inconsistently with the agent's time preferences.

Assume that an agent is automatically enrolled into an imperfect pension scheme, with possibility to opt out prior to period 1. The following proposition characterises when the agent finds the automatic enrolment welfare-improving and thus does not opt-out.

Proposition 2 (stay-in condition) *There exists non-negative $B(\psi, \phi)$, decreasing in ψ and increasing in ϕ , such that an agent finds automatic enrolment into an imperfect scheme welfare improving if $B(\psi, \phi) \geq (1 + \delta)\log(\frac{1}{z})$.*

The derivation is provided in Appendix D. In the above, B denotes a difference between the *laissez-faire* level of lifetime utility of the classical agent and the one of an agent characterised by ψ and ϕ . As such, it may be interpreted as illustrating the extent of the agent's self-control problems and the level of cognitive costs.

The positive impact of automatic enrolment on the agent's welfare is twofold. First, an individual does not need to bear the cognitive costs of decision-making. And second, he does not need to exert any self-control in order to save privately. In case of a 'perfect' pension scheme ($z = 1$), the total impact on the individual's welfare is unambiguous - for economic agents with any degree of self-control problems or cognitive costs, the scheme strictly improves welfare. In case of an imperfect pension scheme (i.e. actuarially unfair or inconsistent with the individual's time preferences), parameter z measures the extent of 'imperfections'. The lower z , the more flawed the scheme. Such a scheme, however, might still be welfare-improving if the positive effects outweigh the scheme's imperfections. For a given value of z , the scheme is more likely to be welfare-improving for the agents characterised by either high ϕ or low ψ .

Suppose that the introduction of automatic enrolment into an imperfect pension scheme affects only the (potential) non-savers, as it is the case with the recent reform in the UK.¹¹ Proposition 2 suggests that the same qualitative features that prevent agents from saving privately (i.e. poor self-control and high cognitive costs) make them more likely to perceive the scheme as welfare-improving and not opt-out. In other words, Lemma 2 and Proposition 2 jointly suggest a potential explanation for the default option effect - the agents who are most likely to be affected by the change of the default are at the same time least likely to opt-out.

Under an alternative interpretation, Proposition 2 provides a theoretical justification for the introduction of automatic enrolment. That is because the individuals who are targeted by the reform (non-savers) are at the same time most likely to be made better off.

Regularities consistent with such default option effects are reflected in the ‘Attitudes to Pensions’ survey. Among those eligible for automatic enrolment, as many as 64% had never heard about the reform. However, 68% agreed that it was a good idea, while 70% reported that they were likely to stay in the scheme once enrolled (MacLeod et al. 2012).

Proposition 2 establishes a condition under which automatic enrolment into a pension scheme improves welfare. In addition, one may be interested in the impact of automatic enrolment on total saving, here defined as the sum of discretionary saving and automatically collected contributions. In standard models with no cognitive costs or self-control problems, the two sources of pension wealth are perfectly substitutable, provided that the pension scheme is actuarially fair and does not force an agent to save ‘too much’ (i.e. more than a^C). In this case, the total saving would be invariant to the level of automatically collected contributions. However, this is

¹¹These are the agents for whom $\phi \geq A(\psi)$, as given in Lemma 2.

no longer true in the setting of the presented model, where the introduction of automatic enrolment might either increase or decrease total saving. In order to focus exclusively on saving implications of automatic enrolment, the following proposition restricts attention to actuarially fair pension schemes.

Proposition 3 (**‘discouraged’ saving and ‘forced’ saving**) *Consider an agent whose optimal level of discretionary saving in absence of automatic enrolment and any cognitive costs is given by a^* . Suppose that the agent is automatically enrolled into an actuarially fair pension scheme that collects a contribution of $\tau \in (0, a^*)$. Then there exist increasing $C(\psi)$ and $D(\psi)$, where $C > D$ for any ψ , such that*

1. *For $\phi \geq C > D$, the total saving under automatic enrolment is strictly greater (‘forced’ saving).*
2. *For $C > \phi \geq D$, the total saving under automatic enrolment is strictly smaller (‘discouraged’ saving).*
3. *For $C > D > \phi$, the total saving is unaffected by automatic enrolment.*

Moreover, D is strictly decreasing in τ .

The derivation is provided in Appendix E. In the above, C and D correspond to potential utility gains from engaging in discretionary saving without and with automatic enrolment, respectively. In the first case, an agent would not save privately and thus automatic enrolment ‘forces’ him to save at least the default contribution τ . In the second case, an agent would save more without automatic enrolment as its introduction decreases potential utility gains from discretionary saving. The agent becomes ‘discouraged’ and saves at a low default of τ , foregoing any private saving. In the third case, an agent saves privately irrespective of the presence of automatic

enrolment and his total saving remains unaffected. The possibilities outlined in Proposition 3 might explain why empirical studies report that the impact of introducing automatic enrolment on aggregate saving is often indeterminate (e.g. Choi et al. 2004; Madrian and Shea 2001).

Note that Proposition 3 does not only imply that the impact of automatic enrolment on total saving is ambiguous in general, as an individual may be ‘forced’ to save, but might also become ‘discouraged’ and rely on low default contributions. It additionally suggests that the impact of the default contribution τ itself is ambiguous. On the one hand, greater τ increases automatic saving. On the other hand, greater τ makes the instance of ‘discouraged’ saving more likely.

As highlighted by the literature, introducing automatic enrolment into pension schemes usually has another crucial effect. Namely, it equalises outcomes, such as wealth accumulation and plan participation, across individuals. A corresponding effect is an implication of Proposition 3.

Corollary 1 (variation in saving levels) *Consider a population of agents with distributions of self-control and cognitive costs given by $\psi \sim [\underline{\psi}, \bar{\psi}]$ and $\phi \sim [\underline{\phi}, \bar{\phi}]$ respectively, such that discretionary savings are given by $a^*(\underline{\psi}, \bar{\phi}) = 0$ and $a^*(\bar{\psi}, \underline{\phi}) \equiv a^* > 0$ in absence of automatic enrolment. Suppose that all agents are automatically enrolled into an actuarially fair pension scheme that collects a contribution of $\tau \in [0, a^*]$. Then, the cross-agent variation in saving levels, as measured by the ratio of maximum to minimum total savings*

$$\frac{\tau + a^*(\bar{\psi}, \underline{\phi}, \tau)}{\tau + a^*(\underline{\psi}, \bar{\phi}, \tau)}$$

is monotonically decreasing in τ and converging to 1.

Corollary 1 is a direct consequence of Proposition 3 and illustrates a potential mechanisms through which the introduction of automatic enrolment equates savings outcomes across self-control and cognitive-cost types. Intuition behind this result is as follows. As the default contribution rate τ increases, agents from the top of the savings distribution either become ‘discouraged’, or maintain their total saving. Agents from the bottom of the distribution, on the other hand, become ‘forced’ to save more, and thus the disparity in wealth accumulation becomes reduced. It disappears altogether when the default contribution rate is equal to the maximum level of discretionary saving observed in the population in absence of automatic enrolment.

1.4 A numerical illustration

This section presents a stylised numerical application of the model. The purpose is to illustrate the above theoretical results in a specific context, and to provide some additional intuition. The numerical exercise is performed as follows. For three income profiles (w_1, w_2, b) , corresponding to 25th, 50th, and 75th percentiles of the earnings distribution in the UK, I assume a distribution of values of the self-control parameter ψ to illustrate the self-control problems of various magnitudes and to match some stylised facts about the impact of self-control problems on wealth accumulation. The parameter ϕ is then selected to match the proportions of non-savers within the particular income categories.¹²

First, I derive and discuss the optimal behaviour for each combination of income

¹²Naturally, there are alternative ways of performing such a numerical exercise. One could, for example, use the existing estimates of measures of self-control or cognitive costs that could be mapped into parameters ψ and ϕ . Alternatively, one may consider using costs of financial advice as an estimate of ϕ . However, I am not aware of any numerical estimates from the existing literature that would correspond reasonably well to the set of parameters employed in the model, its context, and its assumptions regarding the economic environment and functional forms.

and self-control parameter. Subsequently, I analyse three hypothetical reforms of a pension system. Reform 1 introduces automatic enrolment into an imperfect pension scheme. The numerical results suggest that a non-saver with median income would be willing to forego an equivalent of almost 10% of lifetime earnings in order to participate in the scheme. Among those who save, low-income individuals would accept larger losses associated with participation in the scheme. However, the reverse is true for non-savers. Reform 2 introduces automatic enrolment into a fair pension scheme with age-dependent contributions in order to examine the impact of automatic enrolment on aggregate saving. The resulting relationship between aggregate saving and the default contribution rate is approximately U-shaped. This suggests that automatic enrolment paired with low contribution rates may lead to a reduction in aggregate saving, while automatic enrolment with high enough contribution rates increases aggregate saving. Finally, Reform 3 introduces automatic enrolment into a fair scheme with constant contribution rates, expressed as a percentage of agents' income. Because contributions are not tailored to the individual time preferences, such a scheme may decrease welfare. The results indicate that the maximum aggregate welfare is achieved for a contribution rate corresponding to the minimum default required under the UK law. Another stylised fact, namely that the introduction of automatic enrolment reduces variation in accumulated savings, is also replicated.

Consider a three-period version of the model. For $t \in \{1, 2\}$, the instantaneous utility function at t is given by:

$$U_t = \log(C_t) - \Psi(a_t, \alpha_t),$$

where:

$$\Psi(a_t, \alpha_t) = \begin{cases} \psi(\alpha_t - a_t)^2 & \text{for } a_t \leq \alpha_t \\ 0 & \text{for } a_t > \alpha_t \end{cases}$$

and the budget constraint is $C_t \leq w_t - a_t$, as an individual cannot borrow or withdraw his retirement savings. In the last period, $U_3 = \log(C_3)$, where $C_3 = b + (1+r)^2 a_1 + (1+r)a_2$. As in the baseline model, I assume a dual structure of preferences (Benhabib, Bisin 2005; Brocas, Carillo 2008; Fudenberg, Levine 2006, 2012). For $t \in \{1, 2\}$, a doer maximises U_t subject to the budget constraint, while a planner decides whether or not to set the savings targets α_t , which induces some saving by the doer but is (cognitively) costly. The planner's objective is to maximise the lifetime utility function net of the cognitive costs:

$$U = U_1 + \delta U_2 + \delta^2 U_3 - \Phi(\alpha_1, \alpha_2)$$

where

$$\Phi(\alpha_1, \alpha_2) = \begin{cases} \phi & \text{if } \alpha_1 > 0 \\ \delta\phi & \text{if } \alpha_1 = 0, \alpha_2 > 0 \\ 0 & \text{if } \alpha_1 = \alpha_2 = 0 \end{cases}$$

Suppose that an individual is 25 years old in period 1, 45 years old in period 2, and 65 years old when he retires in period 3. Consider three income profiles:

- a ‘low’ income profile with $w_1^L = 16,750$, $w_2^L = 21,881$, and $b = 8,048$,
- a ‘median’ income profile with $w_1^M = 21,962$, $w_2^M = 31,193$, and $b = 8,048$,
- and a ‘high’ income profile with $w_1^H = 28,809$, $w_2^H = 43,592$, and $b = 8,048$.

In case of the median income profile, wages w_1^M and w_2^M are equal to median annual earnings of full-time employees in the UK economy within the age categories

of 22-29 and 40-49, respectively (measured in pounds).¹³ Similarly, the low income profile corresponds to the 25th percentile of the earnings distribution, and the high income profile to the 75th percentile, within the corresponding age categories. The earnings data comes from the Office for National Statistics. Pension benefit b corresponds to a flat-rate New State Pension that was introduced in the UK in 2016. Its value is adjusted to account for the fact that life expectancy at the age of 65 is not exactly 20 years. Moreover, as it is the case with New State Pension, public pension benefit b is independent of the income profile. For a more detailed outline of data sources and remaining assumptions see Appendix G.¹⁴

In the remaining part of this section, assume that economic agents with median income profile constitute 50% of a whole population, while those with low and those with high income profile constitute 25% each.

In line with the standard parameter values used in the literature, set r equal to 0.04 per year and set δ equal to 0.97 per year (e.g. Scholz et al. 2006). For these parameter values, consumption smoothing is achieved by saving in both periods 1 and 2, independently of the income profile.

Assuming away the impact of cognitive costs for the moment, five values of ψ are selected so as to illustrate self-control problems of various magnitudes. Suppose that the distribution of self-control types is uniform and common for all income categories. That is, independent of his income, an individual is equally likely to be characterised by each of five values of $\psi \in \{+\infty, 3 \times 10^{-8}, 1.25 \times 10^{-8}, 6 \times 10^{-9}, 4.3 \times 10^{-9}\}$. For

¹³It is implicitly assumed that the ranking of earnings remains unchanged over the life cycle, i.e. the second-period income of an employee with median income in period 1 is equal to the median income in period 2.

¹⁴Note that the high income profile (as compared to the median) and the median income profile (as compared to the low) are characterised not only by higher levels of earnings, but also by higher growth rates of earnings between periods 1 and 2. While the earnings growth rate is equal to 51% for the high income profile, it is equal to 42% for the median, and 31% for the low profile. This is consistent with a stylised fact in labour economics stating that incomes of high-earnings individuals are increasing at higher rates.

this distribution and the median income profile, the average deviation from the ‘classical’ levels of saving (averaged across periods and self-control levels) is equal to 28%. For the low income profile, the average deviation due to self-control problems is 67%, while for the high income profile it is 21%. This suggests that the self-control problems distort savings behaviour of low-income individuals to a greater extent. Only a few studies estimate the quantitative impact of self-control problems on wealth accumulation (in the field). This varies from 20% to 37% (Ameriks et al. 2007; Schlafmann 2013), which suggests that an average impact of 28%, as in the case of a median earner, can be interpreted as plausible.

Of course, there exists a single value of ψ such that the deviation from the ‘classical’ levels of saving is exactly equal to 28% for a median earner. However, assuming a range of values for ψ allows for populations of economic agents within each income category to consist of both savers and non-savers (see below). Such a distinction would not be possible if all individuals with a given income profile were characterised by a single value of the self-control parameter and the same cognitive cost.

Recall that an economic agent does not save privately when the cognitive costs exceed utility gains from consumption smoothing. Although a large body of evidence is consistent with the notion that the cognitive costs of retirement-related decision-making are indeed non-trivial and have a significant impact on behaviour (e.g. Agnew 2006; Benartzi, Thaler 2004; Binswanger 2010; Choi et al. 2011; Lusardi et al. 2017; Lusardi, Mitchell 2007; Madrian, Shea 2001; Poterba et al. 1996), these studies do not indicate what might be considered a plausible value for ϕ . For that reason, I refer to the ‘Attitudes to Pensions’ survey. The survey reports that as many as 19% of all individuals do not accumulate any resources for retirement. While this proportion is equal to 37% for the respondents with the lowest income, it is only 3% for those from the highest income category. Suppose that the cognitive costs

are independent of the income profile. Then, for any $\phi \in (0.0034, 0.0080)$, 20% of median-earners would not save, 40% low-earners would not save, and all high-earners would save. Therefore assume $\phi = 0.005$, which allows not only to nearly match the percentage of non-savers in the whole population, but also within the income categories. Moreover, for ϕ equal to 0.005, the resulting cognitive costs are not excessive - they correspond in magnitude to a loss of 0.27% of lifetime income by either of the agent-types.¹⁵

Table 1.4.1 reports the optimal levels of saving and resulting utility gains from consumption smoothing, before a deduction of the cognitive costs (measured in utils and denoted GCS).

Table 1.4.1: Saving levels and gains from consumption smoothing

	Low			Median			High		
ψ	a_1	a_2	GCS	a_1	a_2	GCS	a_1	a_2	GCS
$+\infty$	1,304	6,335	0.1212	1,373	9,668	0.1863	1,421	14,073	0.2577
3×10^{-8}	1,065	5,845	0.0735	1,184	9,289	0.1593	1,276	13,783	0.2424
1.25×10^{-8}	798	5,259	0.0080	956	8,807	0.1219	1,089	13,399	0.2212
6×10^{-9}	0	0	0	608	8,025	0.0540	783	12,744	0.1822
4.3×10^{-9}	0	0	0	0	0	0	579	12,289	0.1529

Non-savers are the agents for whom the utility gains from consumption smoothing are smaller than the assumed cognitive costs, i.e. $GCS \leq \phi = 0.005$. For those individuals the reported levels of saving are equal to zero. The above table illustrates the following regularities. First, for any given level of self-control, individuals with higher incomes do not only save more, but also attain larger utility gains from consumption smoothing. As a consequence, higher cognitive costs would have to be assumed in order to prevent a high-income individual from saving, conditional on ψ . Second, within each income category, better self-control implies higher optimal levels

¹⁵Other parameter values for ϕ , that is 0.0034, 0.0065, and 0.008, correspond to lifetime income losses of 0.18%, 0.35% and, 0.43%, respectively.

of saving and resulting lifetime utility. That is because economic agents characterised by better self-control are capable of smoothing their consumption to a fuller extent without bearing an excessive cost of self-control. This is a direct consequence of Lemmas 1 and 3.

1.4.1 Reform 1 - An imperfect pension scheme

In order to analyse the corresponding welfare effects, consider the introduction of automatic enrolment into an imperfect pension scheme. In this case, the variable of interest is the threshold value of z (scheme's 'fairness'), for which an agent would be indifferent between participating in a scheme and saving privately.

A full set of numerical results is reported in Table 1.5.1, Appendix G. Consistent with the theoretical prediction, in case of individuals with perfect self-control, the threshold level of fairness of a pension scheme reflects only the cognitive costs associated with discretionary saving. Second, for any given income profile, the threshold z is (weakly) increasing in ψ . This is due to the fact that economic agents with poorer self-control accept greater imperfections of a scheme, precisely because it enables them to overcome the sizeable self-control problems, as stated in Proposition 2. Among savers, high-earners are less willing to accept scheme's imperfections, because self-control problems distort their savings behaviour to a lesser extent. However, the opposite is true for non-savers as those with lower incomes are less willing to accept unfair schemes. This reversal is due to the fact that economic agents with higher incomes obtain larger utility gains from saving.

In terms of the numerical values, economic agents with perfect self-control require the highest level of fairness of a scheme, their threshold z being equal to 0.997. A median non-saver is willing to accept imperfections corresponding to a loss of almost 10% of his lifetime earnings ($z = 0.904$), while a non-saver with low income is willing

to accept a loss of about 6% ($z = 0.936$). Among savers with imperfect self-control, the marginal z varies across income categories and self-control levels, ranging from 0.928 to 0.989. Arguably, these values indicate that even pension schemes with low levels of fairness, which effectively deprive individuals of between 2% and 10% of their lifetime income, are potentially welfare-improving for a large fraction of agents. These results also highlight why non-savers, who have the lowest threshold z within each income category, are most likely to perceive automatic enrolment as welfare-improving or, in other words, why is there the default option effect.

1.4.2 Reform 2 - Automatic enrolment with age-dependent contribution rates

Consider an introduction of automatic enrolment into an actuarially fair pension scheme with contributions tailored to individual time preferences. Under different scenarios, a given percentage of the ‘classical’ saving levels is collected automatically, relieving the agents from bearing the cognitive costs and exerting self-control.¹⁶ Such construction implies that Reform 2 is unambiguously welfare-improving. For that reason, the emphasis is put on its implications for savings behaviour.

Figure 1.4.1 plots aggregate savings as a function of the contribution level induced by automatic enrolment. Parameter σ denotes a fraction of the ‘classical’ levels of saving that are automatically collected from a participating individual. Aggregate saving is defined as a sum of discretionary saving and automatically collected contributions, averaged over income categories, self-control types, and periods.

As implied by Proposition 3, automatic enrolment has an ambiguous effect on aggregate saving in general. On the one hand, automatic enrolment can increase

¹⁶Recall that irrespective of the level of self-control, all economic agents with given income are characterised by the same levels of saving that maximise lifetime utility.

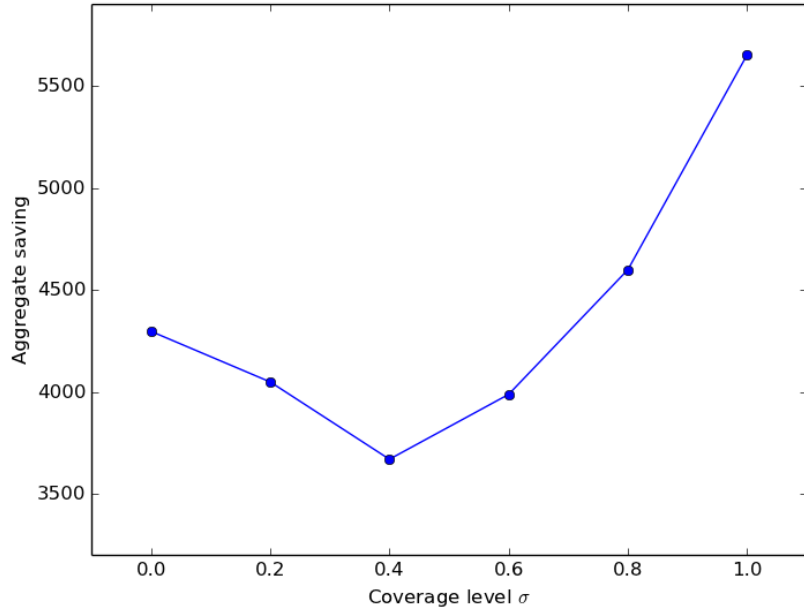


Figure 1.4.1: Default contributions and aggregate saving

aggregate saving by ‘forcing’ non-savers (or savers with poor self-control) to save (save more than before). On the other hand, automatic enrolment reduces gains from smoothing consumption further, which may lead to some economic agents becoming ‘discouraged’ and not saving privately. Figure 1.4.1 illustrates that the aggregate saving is initially decreasing in the level of default contributions, suggesting that the second effect dominates in that region. In other words, automatic enrolment with low default contribution rates decreases aggregate saving. For high enough default contributions, however, the first effect dominates and aggregate saving is increasing until it reaches its maximum at $\sigma = 1$. Automatic enrolment with high default contributions increases aggregate saving by not only compelling non-savers to save, but also, and more importantly, by compelling them to save enough. This results in a U-shaped relationship between the measure of aggregate saving and σ .

1.4.3 Reform 3 - Automatic enrolment with constant contribution rates

As a final numerical application, consider an introduction of automatic enrolment into an actuarially fair pension scheme with constant contribution rates, expressed as a percentage of agents' earnings. This intervention is more realistic than Reform 2, as policymakers are rarely able to infer individual time preferences and cater to them exactly. Suppose that every period a given percentage of earnings, denoted τ , is automatically contributed towards a future pension benefit. As opposed to Reform 2, such an intervention allows for levels of contributions to be inconsistent with the individual time preferences, but it is more in line with the actual regulations of workplace pensions. Table 1.4.2 reports the measures of aggregate welfare and aggregate saving for a range of values for τ , assuming that economic agents do not opt out of the scheme. The measures are normalised to 1 at $\tau = 0\%$ and averaged over income and self-control type.

Table 1.4.2: Automatic enrolment with constant contribution rates

τ	0%	2%	4%	6%	8%	10%	12%	14%
Welfare	1.00000	1.00090	1.00198	1.00269	1.00274	1.00266	1.00241	1.00195
Saving	1.000	0.827	0.712	0.806	0.786	0.754	0.802	0.885

Automatic enrolment into pension schemes may make economic agents better off by allowing them to overcome their self-control issues and counteract high cognitive costs. At the same time, a pension scheme with imperfectly tailored contribution rates may decrease welfare if it compels the agents to save 'too much' or 'at wrong times'. In the current example, the measure of aggregate welfare follows an inverted U-shaped pattern, which indicates that the positive welfare effects prevail for lower values of τ , while the negative effects dominate for higher values. Interestingly, the maximum aggregate welfare is achieved for $\tau = 8\%$, which is equal to the minimum

default contribution rate into workplace pension schemes required under the UK law.¹⁷

Moreover, even for the welfare-improving contribution rates ($\tau \leq 8\%$), one can observe heterogeneous effects across self-control types. While individuals characterised by good self-control are made worse off by the scheme, those with poor self-control are made better off. This illustrates that the welfare implications of introducing automatic enrolment into pension schemes depend not only on the selected default, but also on the distribution of self-control types in the population.

Aggregate saving is non-monotonous in the default contribution. For instance, for contribution rates of 6% and higher, the borrowing constraint in period 1 binds for almost all agent-types and there is no discretionary saving taking place on an early stage of the working life.

The numerical results highlight another important consequence of automatic enrolment. Within all income categories, higher values of τ unambiguously lead to lower variation in saving and lifetime utility across the self-control types. As the contribution rate increases, ratios of saving and lifetime utility obtained by agents with good and poor self-control decrease monotonically and converge to 1, as implied by Corollary 1. The corresponding numerical values are reported in Appendix G.

1.5 Conclusion

The primary goal of this chapter was to advocate the relevance of accounting for cognitive costs in retirement-related decision-making. The resulting possibility of corner solutions ('rational inaction') may explain the observed patterns of savings

¹⁷Following the full implementation of the automatic enrolment reform, from April 2019, the default contribution rate cannot be lower than 8% of employee's wage, out of which at least 3% has to be the employer's contribution.

behaviour, in particular consequences of introducing automatic enrolment into pension schemes. As such, this study might be seen as an initial step towards incorporating cognitive costs into more advanced theoretical and quantitative models of saving.

Certainly, the scope of the above analysis is limited in a few important ways. First, the assumed functional forms were introduced for simplicity and tractability. Predictions of the model were therefore stated mainly in qualitative terms, and the simple numerical application of Section 4 was performed in order to provide additional intuition behind the mechanics of the model. Second, the analysis focused exclusively on a case of a ‘sophisticated’ individual, who is perfectly aware of the degree of his financial self-control. A version of the model that does not assume full sophistication about the ability to exercise self-control, akin the model of procrastination by O’Donoghue and Rabin (1999a, 2001), would yield different predictions and policy recommendations. For example, while forbidding delays in action-taking or simplifying the decision environment would improve the outcomes under the model of procrastination due to naiveté about one’s self-control problem, it could be detrimental to welfare in the model of rational inaction. Third, the presented model abstracts from an array of factors that have a significant impact on retirement planning, such as uncertainty, possibility to borrow during one’s working life, or timing of retirement. This list is by no means exhaustive. Extending the analysis and improving our understanding of the impact of cognitive costs on financial decisions should be the objectives of future work.

Appendix

Appendix A - Decision-making of a doer

In period 1, the doer's optimisation problem is:

$$\max_{C_1, a} \log(C_1) - \psi(\alpha - a)^2$$

subject to $C_1 \leq w - a$. Equivalently:

$$\max_a \log(w - a) - \psi(\alpha - a)^2$$

The first order condition (FOC) is satisfied at two points:

$$a = \frac{w + \alpha \pm \sqrt{(w - \alpha)^2 + \frac{2}{\psi}}}{2}$$

However, the second order condition for a maximisation problem is satisfied only at:

$$a = \frac{w + \alpha - \sqrt{(w - \alpha)^2 + \frac{2}{\psi}}}{2}$$

Thus one obtains a unique solution for a^* . Accounting for the no-borrowing constraint gives:

$$a^* = \max \left\{ \frac{w + \alpha - \sqrt{(w - \alpha)^2 + \frac{2}{\psi}}}{2}, 0 \right\}$$

It is straightforward to notice:

$$\frac{\partial a^*}{\partial \psi} \geq 0, \quad \frac{\partial a^*}{\partial w} \geq 0, \quad \frac{\partial a^*}{\partial \alpha} \geq 0, \quad \lim_{\psi \rightarrow \infty} a^* = \alpha,$$

Moreover, for a given savings target α , the cost of exercising self-control is decreasing in ψ :

$$\begin{aligned} \frac{d\psi(\alpha - a^*)^2}{d\psi} &= (\alpha - a^*)^2 - 2\psi(\alpha - a^*) \frac{\partial a^*}{\partial \psi} \leq 0 \iff \\ &\iff \frac{\sqrt{(w_1 - \alpha_1)^2 + \frac{2}{\psi}}}{(w_1 - \alpha_1) + \sqrt{(w_1 - \alpha_1)^2 + \frac{2}{\psi}}} \leq 1 \end{aligned}$$

which is true.

□

Appendix B - Decision-making of a planner

When the planner does not set a savings target, she attains the lifetime utility of $\mathcal{U} \equiv \log(w) + \delta \log(b)$. If, on the other hand, she sets the target, she attains:

$$\mathcal{V} - \phi \equiv \max_{\alpha} \{ \log(w - a^*) - \psi(\alpha - a^*)^2 + \delta \log(b + (1 + r)a^*) \} - \phi$$

At the optimum, the lifetime utility given by $\max \{ \mathcal{U}, \mathcal{V} - \phi \}$ is (weakly) decreasing in ϕ . Moreover, it is also (weakly) increasing in ψ :

$$\frac{d\mathcal{V} - \phi}{d\psi} = \underbrace{\frac{\partial \mathcal{V} - \phi}{\partial a^*}}_{=0} \frac{\partial a^*}{\partial \psi} + \frac{\partial}{\partial \psi} \left[\log(w - a^*) - \frac{1}{4\psi(w - a^*)^2} + \delta \log(b + (1 + r)a^*) \right] \geq 0$$

Naturally, $\mathcal{V} - \mathcal{U} \geq 0$ due to the fact that a lack of a savings target imposes $\alpha = 0$. Define $A \equiv \mathcal{V} - \mathcal{U}$. The planner optimally sets no saving target when $\phi \geq A$. Following from the same property for \mathcal{V} , A is increasing in ψ .

Now consider the derivation of Lemma 3. Assume that incomes w and b are such that the classical agent would save in period 1. The classical agent chooses his level of savings a^C by solving:

$$\max_a \log(w - a) + \delta \log(b + (1 + r)a)$$

and thus a^C satisfies the FOC:

$$\delta \frac{1+r}{b+(1+r)a} = \frac{1}{w-a}$$

An agent with $\psi < +\infty$, on the other hand, solves:

$$\max_{\alpha} \log(w - a^*) - \psi(\alpha - a^*)^2 + \delta \log(b + (1 + r)a^*)$$

and the optimal saving satisfies:

$$\delta \frac{1+r}{b+(1+r)a^*} = \frac{1}{w-a^*} + 2\psi(\alpha - a^*)\left(\frac{1}{\frac{\partial a^*}{\partial \alpha}} - 1\right)$$

The last term on the right-hand side is positive because $\frac{\partial a^*}{\partial \alpha} \leq 1$. Thus $a^*(\alpha^*) \leq a^C$.

It is also straightforward to show that this last term is decreasing in ψ and converges to zero as ψ increases. This implies that $a^*(\alpha^*)$ is increasing in ψ and converges to a^C .

□

Appendix C - Proposition 1

In case of the classical agent, a^C solves:

$$\max_a \log(w - a) + \delta \log(b + (1 + r)a)$$

which yields the following solution:

$$a^C = \frac{\delta}{1+\delta}w - \frac{1}{(1+\delta)(1+r)}b$$

At the optimum, the relationship between a^C and b is linear and therefore discretionary saving and public pension wealth are perfect substitutes.

Now assume positive cognitive costs $\phi > 0$, but no self-control problem. If an agent saves privately, the optimal level of savings is again a^C . Let:

$$\mathcal{U} \equiv \log(w) + \delta \log(b)$$

$$\mathcal{V} \equiv \log(w - a^C) + \delta \log(b + (1 + r)a^C)$$

The agent saves as long as $\mathcal{V} - \mathcal{U} > \phi$. Notice that $\mathcal{V} - \mathcal{U}$ is decreasing in b :

$$\frac{d\mathcal{V}-\mathcal{U}}{db} = \underbrace{\frac{\partial \mathcal{V}}{\partial a^C}}_{=0} \frac{\partial a^C}{\partial b} + \frac{\delta}{b+(1+r)a^C} - \frac{\delta}{b} \leq 0$$

Thus for high enough b , $\mathcal{V} - \mathcal{U} \leq \phi$, in which case the agent does not save, $a^* = 0$. Denote the parameter value at which $\mathcal{V} - \mathcal{U} = \phi$ by \hat{b} . Introduction of positive cognitive costs results in the optimal savings being non-linear in b :

$$a^* = \begin{cases} \frac{\delta}{1+\delta}w - \frac{1}{(1+\delta)(1+r)}b & \text{for } b < \hat{b} \\ 0 & \text{for } b \geq \hat{b} \end{cases}$$

Now introduce self-control problems, ($\psi < +\infty$). The optimisation problem yields the following FOC:

$$\delta \frac{1+r}{b+(1+r)a^*} = \frac{1}{w-a^*} + 2\psi(\alpha - a^*)\left(\frac{1}{\frac{\partial a^*}{\partial \alpha}} - 1\right)$$

and the resulting $a^*(\alpha^*)$ is non-linear in b .

Consequently, self-control problems paired with non-zero cognitive costs result in $a^*(\alpha^*)$ being non-linear and discontinuous in b .

□

Appendix D - Proposition 2

Let $U^C \equiv \log(w - a^C) + \delta \log(b + (1 + r)a^C)$. Then automatic enrolment into an imperfect pension scheme allows an agent achieve lifetime utility of $U^C + (1 + \delta)\log(z)$, whereas without automatic enrolment, she attains $\max \{\mathcal{U}, \mathcal{V} - \phi\}$, where:

$$\mathcal{U} \equiv \log(w) + \delta \log(b)$$

$$\mathcal{V} - \phi \equiv \max_{\alpha} \{\log(w - a^*) - \psi(\alpha - a^*)^2 + \delta \log(b + (1 + r)a^*)\} - \phi$$

The agent finds automatic enrolment welfare improving when $B(\psi, \phi) > (1 + \delta)\log(\frac{1}{z})$, where $B(\psi, \phi) \equiv U^C - \max \{\mathcal{U}, \mathcal{V} - \phi\}$ is non-negative, (weakly) decreasing in ψ , and (weakly) increasing in ϕ .

□

Appendix E - Proposition 3

Without automatic enrolment, an agent saves if $C > \phi$, where $C(\psi) = \mathcal{V} - \mathcal{U}$ represent the utility gain from saving privately, as defined in Appendix B. In this case, discretionary (total) saving is equal to $a^*(\alpha^*)$. Otherwise it equals 0.

Under automatic enrolment, the agent saves if $D > \phi$, where $D(\psi)$ represents the utility gain from saving privately after a contribution of $\tau \in (0, a^*)$ has been collected. In this case, the total saving is given by $\hat{a}^*(\hat{\alpha}^*) + \tau = a^*(\alpha^*)$. Otherwise it equals τ .

Note that both C and D are increasing in ψ , but because $\tau > 0$, $C > D$ for any ψ . Moreover, D decreases in τ . This follows from the derivation of Proposition 1 (Appendix C).

□

Appendix F - Numerical illustration

Currently, the State Pension Age in the UK is equal to 65 for men and women. According to the Office for National Statistics (ONS), the life expectancy of a 65 year old woman is 20.88 and that of a 65 year old man is 18.32.¹⁸ Assuming equal weights, an average individual is expected to live for 19.6 more years at the age of 65. For that reason assume that periods 1 and 2 correspond to 20-year-long time intervals. An economic agent is then 25 years old in period 1, 45 years old in period 2, and retires at the age of 65 in period 3.

According to the Annual Survey of Hours and Earnings conducted by ONS, in 2015 median annual pay of all full-time employees was equal to £21,962 within the age category of 22-29, and £31,193 within the age category of 40-49. In case of the 25th percentile of the earnings distribution, these incomes are equal to £16,750 and £21,881, within the respective age categories. In case of the 75th percentile, these incomes are equal to £28,809 and £43,592, within the respective age categories.¹⁹

The New State Pension (a single-tier benefit) provides future retirees with a pension benefit equal to £151.25 per week or £7887 per year.²⁰ Scaling this number in order to account for the fact that retirement period is expected to last slightly shorter than 20 years results in $b = 8048$.

The interest rate corresponding to a real rate of return of 0.04 per annum can be seen as a standard value used in the literature (e.g. Binswanger 2012; Caliendo, Gahramanov 2013; Scholz et al. 2006). Thus I set $(1 + r) = (1.04)^{20} = 2.19112$.

In general, there is no consensus in the empirical literature about the ‘correct’ size of the intertemporal discount factor. Although many researchers use a value of

¹⁸Source: <http://www.ons.gov.uk/ons/taxonomy/index.html?nscl=Ageing#tab-data-tables>. Accessed: 16/09/2015.

¹⁹Source: <http://www.ons.gov.uk/ons/publications/re-reference-tables.html?edition=tcn%3A77-400803>. Accessed: 18/11/2015.

²⁰Source: <https://www.gov.uk/new-state-pension/overview>. Accessed: 16/09/2015.

0.96 per year as a standard value (e.g. Binswanger 2012; Scholz et al. 2006; Winter et al. 2012), the empirical estimates of the discount factor vary from values as low as 0 to values significantly above 1, depending on the context and methodology (Frederick et al. 2002). Keeping these caveats in mind, I set $\delta = (0.97)^{20} = 0.54379$.

Table 1.5.1 reports values for variable z for which an agent would be indifferent between participating in a scheme and saving privately. Note that the underlying parameter values span savers as well as non-savers. These threshold values for z for the subgroup of non-savers are reported in italics.

Table 1.5.1: Threshold z

	Low	Median	High
ψ	Marginal z	Marginal z	Marginal z
$+\infty$	0.997	0.997	0.997
3×10^{-8}	0.972	0.983	0.989
1.25×10^{-8}	0.938	0.963	0.978
6×10^{-9}	<i>0.936</i>	0.928	0.957
4.3×10^{-9}	<i>0.936</i>	<i>0.904</i>	0.942

Regarding Reform 3, Table 1.5.2 reports the ratios of average savings and lifetime utility attained by an agent with median income and perfect self-control ($\psi = +\infty$) to those obtained by an agent with poor self-control ($\psi = 4.3 \times 10^{-9}$), as a function of τ .

Table 1.5.2: Automatic enrolment and variation in welfare and savings

τ	0%	2%	4%	6%	8%	10%	12%	14%
Welfare	1.00973	1.00585	1.00342	1.00189	1.00093	1.00032	1.00000	1.00000
Saving	$+\infty$	9.504	4.353	2.645	1.929	1.543	1.000	1.000

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Chapter 2. Savings Contracts for Naïve Agents: The baseline model

Recent pension reforms in OECD countries endow individuals with more responsibility for their financial security in retirement, raising concerns about their ability to select appropriate pension arrangements and save adequately. Chapters 2 and 3 analyse the interaction between a present-biased individual and a financial provider in order to examine the properties of equilibrium savings contracts and the impact of common policy interventions. Using a tractable model, in Chapter 2 I find that naïve present-biased agents are offered exploitative contracts that are either ‘inefficiently cheap’ (low-yield, low-fee) or ‘inefficiently expensive’ (high-yield, high-fee), depending on whether the income or the substitution effect of an interest rate change dominates in the agent’s utility function.

2.1 Introduction

Recent pension reforms in the OECD countries tend to reduce the generosity of public benefits and increase the importance of private pensions. Half of the countries had introduced mandatory (or quasi-mandatory) private pensions and achieved

nearly universal coverage of the working-age population.¹ While private pension arrangements are voluntary in the US and the UK, they nonetheless cover over 40% of the population. The growing importance of private pensions as determinants of future retirement incomes is also reflected in the fact that private pension assets, expressed as a percentage of GDP, have been increasing over the last 20 years in all OECD countries. By the end of 2016, private pension assets were worth over \$38 trillion and the US alone accounted for two thirds of that amount. What is especially important is the fact that these private pension arrangements are predominantly of the defined-contribution (DC) type, under which assets accumulated by the time of retirement directly determine the amount of pension benefit. That implies that individuals bear most of the risks associated with accumulation (e.g. investment performance, employment) and decumulation (e.g. longevity) stages of retirement saving (OECD 2016, 2017).

Given the changing policy environment, a natural question arises of whether individuals are capable of selecting an appropriate pension arrangement and preparing themselves financially for retirement. Note that these choices, characterised by high degrees of complexity, intertemporal nature of trade-offs, long planning horizons and infrequent feedback, provide little opportunity for learning by doing. Indeed, rich empirical evidence suggests that wealth accumulation is significantly affected by self-control problems (Ameriks et al. 2007; Ashraf et al. 2006) and procrastination or inattention (Benartzi, Thaler 2004, 2007; Chetty et al. 2014; Choi et al. 2004, 2011; Madrian, Shea 2001). These behaviours can be explained by a model of present-biased preferences (or: hyperbolic discounting), which has been used to generate undersaving (Laibson 1994, 1997; Diamond, Kőszegi 2003) and procrastination

¹Quasi-mandatory pensions are established via industry- or nation-wide collective bargaining agreements and employees are obliged to participate in such schemes. The coverage of quasi-mandatory pensions has reached 90% of the working-age population in Sweden, 88% in the Netherlands, and 63% in Denmark, for example (OECD 2017).

(O'Donoghue, Rabin 1999a, 1999b, 2001), as well as to analyse a socially optimal choice of commitment and default options in pension systems (Beshears et al. 2014; Choi et al. 2003).²

Market interactions between an individual and a providing firm are nonetheless relatively understudied. The prevailing approach is to analyse the biased consumption-saving decisions without considering the supply side of the market for financial products, and thus to treat the alternatives available to an agent as exogenously given, or provided by a social planner. It appears important, however, to account for the incentives of private providers as well as predict the response of the market to various policy interventions. Numerous policy reports highlight the need for appropriate regulation of markets for pension products (e.g. OECD 2016, 2017; Office of Fair Trading 2014). In order to analyse the properties of endogenously determined savings contracts, I model the market interaction between a financial provider and a present-biased individual. In Chapter 2, I analyse a simplified model, which allows for analytical tractability and provides helpful intuition. When the rate of return on accumulated savings determines the agent's valuation of a contract, the exploitative contracts aimed at naïve present-biased individuals are either 'inefficiently cheap' (low-yield, low-fee) or 'inefficiently expensive' (high-yield, high-fee), depending on whether the income or the substitution effect of an interest rate change dominates in the utility function. In Chapter 3, I present a multiperiod model and embed the interaction with a pension provider in a rich life-cycle framework with hyperbolic discounting. This approach allows to assess the quantitative importance of contractual design and choice in a more realistic environment, which extends the scope of the exploitative contracting literature. The results indicate the prevalence of inefficient contracts in the market, which generates a small, but non-trivial loss of consumer

²Goda et al. (2018) provide evidence for an empirical link between a direct measure of present bias and retirement savings.

welfare of 0.17% per annum.

In the simple model, the individual first decides whether or not to sign a contract proposed by the provider, and subsequently chooses his level of saving. In the baseline case, a monopolistic provider offers contracts that specify a rate of return on the agent's savings and a flat-rate fee charged for the service.³ When the provider can observe the individual's type and tailor the contract terms accordingly, contracts offered to those agents who are aware of their future present bias ('sophisticates') are efficient in the sense that they maximise the consumer-firm surplus. On the other hand, contracts offered to those agents who are not fully aware of their future present bias ('naifs') are designed to exploit their naiveté and thereby increase the provider's profits.⁴ Importantly, these exploitative contracts are inefficiently cheap when the income effect of an interest rate change dominates in the agent's utility function, but inefficiently expensive when the substitution effect dominates.⁵ Empirically, the income effect appears to be stronger for an average consumer, which implies the prevalence of low-yielding exploitative contracts (Attanasio, Weber 2010).

To provide intuition for this result, note that naïve present-biased agents are overly optimistic about their future saving behaviour and thus their willingness to pay for any given contract exceeds the willingness to pay of a sophisticated agent. This could lead to the hasty conclusion that naifs, relative to sophisticates, will be

³Investment performance and associated charges are arguably the most important features of a private pension arrangement (Office of Fair Trading 2014). As the model abstracts from financial risk, 'offering a higher interest rate' is interpreted as illustrating the provider's decision to offer active portfolio management or a wider range of investment options.

⁴'Naifs' are not only agents who are completely unaware of their present bias, but also those who underestimate the magnitude of the bias, i.e. 'partially naïve' individuals as defined by O'Donoghue and Rabin (2001).

⁵In general, the response of optimal savings to changes in the interest rate is ambiguous. On the one hand, a higher interest rate makes future consumption cheaper relative to current consumption. This generates the substitution effect, according to which optimal savings would increase in the interest rate. On the other hand, under a higher interest rate a lower level of saving is required in order to achieve any given level of future consumption. This gives rise to the income effect, according to which optimal savings would decrease in the interest rate. Whichever of these two effects is stronger is determined by the curvature of the utility function.

offered more expensive savings products with generous rates of return. It turns out that the distortion is more subtle than that. Rather, the profit-maximising provider exploits the naiveté about the present bias by offering contract terms that appear attractive as long as the individual saves a lot. This is achieved by distorting the interest rate downwards when the income effect dominates and upwards when the substitution effect dominates.⁶ This novel result highlights the importance of interactions between ‘behavioural’ and ‘classical’ preference parameters in generating predictions.

Naïve agents who sign exploitative contracts suffer welfare losses, as compared to their sophisticated counterparts. These welfare losses can be decomposed into the ‘distributional effect’, according to which naifs would overpay for any given contract, and the ‘efficiency effect’, arising from the fact that the offered contract terms are inefficiently distorted. A providing firm generates higher profits when interacting with naifs. However, the agent’s present bias leads to consumer welfare losses, inefficiency, and higher profits only when coupled with naiveté.

Given the consumer exploitation, I consider three policy interventions in the market for pension products which have already been implemented in some OECD countries. First, imposing restrictions on fees charged by the provider, results in (weakly) lower rates of return being offered to all agent-types. An effective cap on fees thus distorts otherwise efficient contracts designed for sophisticated agents and might further reduce both efficiency and consumer welfare for naifs if their exploitative contracts were ‘too cheap’ in the first place. Second, increasing the degree of competition raises consumer welfare while reducing the provider’s profits. How-

⁶To be more explicit about the underlying mechanism, a provider chooses contract terms which magnify the overvaluation of a savings contract by a naif, that is the difference between the agent’s willingness to pay and the contract’s actual worth, which will determine the firm’s cost. The naifs’ tendency to overpay for their contracts is therefore exploited by distorting the interest rate in the direction which induces higher expected savings.

ever, the exploitative features and efficiency properties of savings contracts remain unaffected.⁷ Third, introducing a minimum-savings rule has an ambiguous impact on efficiency and consumer welfare in general. However, close to the case of complete naiveté, the minimum-savings requirement only exacerbates the exploitative contract design, reducing efficiency and consumer welfare. This result follows from the fact that the *laissez faire* exploitative contracts already induce the naïve agents to over-save, relative to sophisticated, present-biased individuals. Taken together, these results emphasize the challenges to policy-making in this domain.

Key implications of the model regarding the exploitative contract design are robust, in qualitative sense, to the degree of competition, variable fees, and financial illiteracy. Another important extension relaxes the assumption of perfect observability of the agent's characteristics. I consider two specific populations of two indistinguishable agent-types. First, a population of agents differing in their present bias, but holding identical beliefs about the magnitude of the bias. Second, a population of agents holding heterogeneous beliefs about their present bias, but characterised by the same magnitude of the bias. In the first case, naïve agents might be either better off or worse off relative to the baseline case where their type is observed. In the second case, naifs are always (weakly) better off relative to the baseline. In general, the efficiency of sophisticates' contracts deteriorates relative to perfect observability, but this is not necessarily compensated for with an improvement in the efficiency of naifs' contracts.

Chapters 2 and 3 contribute to the expanding literature on exploitative contracting (or: behavioural industrial organisation). In a seminal paper, DellaVigna and Malmendier (2004) show that a provider serving individuals with self-control

⁷Analogous remarks can be made for an alternative case of discouraging competition - insofar as these interventions reduce the value of the agent's outside option, they increase provider's profits and decrease consumer welfare, but the efficiency of outcomes remains unaffected.

problems prices investment goods (such as gym entrance) below their marginal cost, and leisure goods (such as credit card debt) above their marginal cost. While such design helps sophisticated agents overcome their self-control problems, it also exploits mispredictions of future usage by naifs. Subsequently, exploitative contracting framework has been applied to study contracts for products with shrouded attributes (Gabaix, Laibson 2006), mobile phone services (Grubb 2009), and credit card debt (Heidhues, Kőszegi 2010).⁸ The current chapter constitutes the first application of the exploitative contracting framework to markets for savings products. Importantly, while DellaVigna and Malmendier (2004) already noted that the direction of exploitative distortions to naifs' contracts is ambiguous, this chapter provides a meaningful condition that determines whether such contracts are 'inefficiently cheap' or 'inefficiently expensive'. This novel theoretical finding highlights the importance of interactions between parameters traditionally seen as 'classical' and 'behavioural' and such considerations have so far received little attention from the behavioural economics literature (Kőszegi 2014). To my knowledge, the numerical application presented in Chapter 3 is the first attempt to explicitly assess the quantitative implications of exploitative contracting. The current approach combines numerical tools from macroeconomics with a theoretical microeconomic work on contract design. Accounting for the supply side of the market for financial products is also novel in the context of the life-cycle literature, both theoretical and applied, as well as 'classical' and behavioural (see e.g. Angeletos et al. 2001; Attanasio, Weber 2010; Gourinchas, Parker 2002; Laibson et al. 2000, 2017).

The remainder of this chapter proceeds as follows. Section 2 introduces the baseline version of the simple model and presents the main results. Section 3 discusses the three government interventions. Section 4 outlines a range of extensions to the

⁸Eliasz and Spiegel (2006) present a more general model of contracting with diversely naïve agents.

baseline model. Section 5 concludes. Derivation of the results and a fuller treatment of the extensions and alternative modelling assumptions are provided in the appendix.

2.2 The baseline model

As a baseline case, consider a market for savings products in which an individual interacts with a monopolistic provider (‘the firm’) who can perfectly observe agent’s characteristics and tailor the contract terms accordingly. A savings contract P is characterised by two unconditional parameters - an interest rate r that the provider offers on agent’s savings and a flat-rate (‘per-period’) fee f that is charged for the service.⁹ There are two stages in the model. In period 0 the contract is proposed by the firm and evaluated by an individual, who then saves in period 1 taking the contract parameters as given. Such decoupling of contract evaluation and saving is supposed to illustrate delays that occur between choosing one’s pension provider and accumulating pension wealth, mainly due to a long horizon of repeated savings decisions.

The agent’s preferences as in period 0 are given by the following quasilinear utility function:

$$U_0 = u(c_1) + \delta u(c_2) - f,$$

where $c_1 = Y - s$, $c_2 = (1 + r)s$, s denotes agent’s savings, $Y > 0$ is exogenous pre-

⁹Investment performance and associated charges are arguably the most important features of a private pension arrangement (Office of Fair Trading 2014). Conditioning those parameters, for instance on the realised level of saving, is discussed in section 2.2.3. Moreover, it must be noted that in reality pension providers or portfolio managers typically do not guarantee a specific rate of return as most investment strategies are inherently risky. In terms of interpretation, one can view the decision to ‘offer higher interest’ as illustrating the provider’s offer of active portfolio management or a wider range of investment options.

retirement income, and $\delta \leq 1$ is the discount factor.¹⁰

In period 1, when saving takes place, an agent is potentially present-biased and acts so as to maximise the following utility function:

$$U_1 = u(c_1) + \beta\delta u(c_2) - f,$$

where $\beta \in [0, 1]$ represents the magnitude of the present bias. Typically, present bias reflects the impact of non-normative factors that make an agent seemingly over-value his current consumption relative to future consumption, such as temptation, inattention or procrastination. The lower β , the larger the self-control problem. Conversely, for $\beta = 1$, the behaviour is not affected by present bias.

Under present bias, the agent's preferences become time-inconsistent. Clearly, a consumption bundle that maximises utility function U_1 for $\beta < 1$ differs from a consumption bundle that maximises U_0 . Following the standard approach in the literature, allow U_0 to be interpreted as a normative benchmark (e.g. O'Donoghue, Rabin 1999a, 2001; Heidhues, Kőszegi 2010; see the related discussion in Spiegler 2014).

Furthermore, note that the beliefs about savings behaviour in period 1 are crucial when evaluating the contract in period 0. Following O'Donoghue and Rabin (2001), denote period-0 singleton beliefs regarding agent's present bias by $\hat{\beta} \in [\beta, 1]$. That is, in period 0 an individual believes that his period-1 self will act so as to maximise

¹⁰There are several reasons why the utility function is linear in the transfer between the agents and the firm. First, I am implicitly assuming that the intertemporal decision-making remains unaffected by the fee. In the context of life-cycle wealth accumulation, where $Y \gg f$, this is not an overly restrictive assumption. Consequently, the interest rate is the only contract parameter that affects the intertemporal trade-off. Second, the efficiency criterion, which is introduced later, is constant in the transfer if the firm is risk-neutral. In particular, efficiency is invariant to 'money changing hands.' Third, the analysis is simplified without a significant loss of insight. As long as the impact of fees on the intertemporal trade-off is 'small', the interest rate is the contract parameter that determines (perceived) consumer surplus, while the fee is used by the provider to extract the surplus. Then, the main implications of the baseline model remain valid. The appendix relaxes the assumption of quasi-linearity.

U_1 parametrised by $\hat{\beta}$. Agents with beliefs $\hat{\beta} > \beta$ are called ‘naïve’ because they underestimate the magnitude of their present bias. The greater $\hat{\beta}$, the more severe the underestimation. On the other hand, agents with beliefs $\hat{\beta} = \beta$ are ‘sophisticated’ in the sense that they are fully aware of their present bias.

For notational simplicity, but without loss of insight, the baseline model sets $\delta = 1$.

Naifs’ overly optimistic beliefs about their future savings behaviour make them overvalue a savings contract P for any given parameters r and f .

Lemma 1 *For a concave $u(\cdot)$, naïve agents overestimate their future saving and thus overvalue any savings contract.*

The short proof is relegated to the appendix. Naturally, more severe naiveté results in greater misprediction of future savings behaviour and overvaluation of a contract.

Assume a CRRA utility formulation, i.e.:

$$u(x) = \frac{x^{1-\theta}-1}{1-\theta},$$

where $\theta > 0$ is the CRRA parameter.¹¹ The CRRA utility function features prominently in the literature on intertemporal choice, especially in the context of a life cycle (see e.g. Attanasio, Weber 2010; Browning, Lusardi 1996; Gourinchas, Parker 2002).

In general, the response of optimal savings to changes in the interest rate is ambiguous. On the one hand, a higher interest rate makes future consumption cheaper relative to current consumption. This generates the substitution effect, according

¹¹CRRA stands for ‘constant relative risk aversion’. Moreover, this utility function is also characterised by a constant elasticity of intertemporal substitution, given by the inverse of θ . A single parameter capturing both risk preferences and elasticity of intertemporal substitution is a strength as well as a limitation of the parsimonious CRRA utility formulation.

to which optimal savings would increase in the interest rate. On the other hand, under a higher interest rate a lower level of saving is required in order to achieve any given level of future consumption. This gives rise to the income effect, according to which optimal savings would decrease in the interest rate. From this perspective, a CRRA utility formulation is particularly useful, as the size of parameter θ determines whether the substitution or the income effect dominates. For $\theta < 1$ the substitution effect is stronger and optimal savings increase in the interest rate. For $\theta > 1$, the income effect dominates and optimal savings decrease in the interest rate. For the special case of $\theta = 1$, the CRRA utility function takes a logarithmic form. Then, the optimal saving is invariant to the interest rate.

To introduce concise notation, let the agent's valuation of a savings contract given his belief $\hat{\beta}$ be represented by \hat{U}_0 :

$$\hat{U}_0 \equiv U_0(s(\hat{\beta}, r), r, f) = \frac{[Y - \hat{s}]^{1-\theta} - 1}{1-\theta} + \frac{[(1+r)\hat{s}]^{1-\theta} - 1}{1-\theta} - f,$$

where $\hat{s} \equiv s(\hat{\beta}, r)$ is the forecasted savings given the contract parameters.¹² Denote the contract valuation net of fees by $\hat{V}_0 \equiv V_0(s(\hat{\beta}, r), r)$, so that $\hat{U}_0 = \hat{V}_0 - f$.

If an agent does not sign a savings contract, he obtains reservation utility of \underline{u} . Under the assumption of monopoly, \underline{u} is interpreted as a level of utility associated with reliance on an exogenously given system of public pension benefits. That is, the more generous (or comprehensive) the state pensions, the greater \underline{u} . Because the reservation utility does not depend on the agent's actions, the baseline model supposes that \underline{u} is constant across agent-types.

Suppose that a savings contract is offered by a financial provider implicitly interpreted to be a pension fund. Most private pension assets are managed by pension

¹²Since $Y > 0$ and $u(\cdot)$ is concave, the maximisation problem of U_1 (or \hat{U}_1) has an interior solution. Thus the borrowing constraint is omitted. Under the assumption of quasi-linearity in fees, the actual and forecasted savings are not affected by f .

funds in the OECD countries (OECD 2017).¹³ Moreover, pension funds are able to offer tailor-made products, both via a retail market and workplace arrangements. Thus the issues of contract design and appropriate regulatory policies are particularly relevant when applied to pension funds. The literature analysing the determinants of associated costs of provision (Basu, Andrews 2014; Bateman, Mitchell 2004; Bikker, de Dreu 2009; Bikker et al. 2012) divides the total cost of a pension fund into administrative and investment costs. There is consistent evidence that the administrative costs are increasing in the size of an individual pension pot, controlling for the number of pension plan participants, and that there is a substantial fixed cost component at the individual level. The available results additionally suggest that the investment costs are increasing in the (expected) rate of return.¹⁴ Consequently, suppose that a savings contract is offered by a provider who maximises the following profit function:

$$\pi = f - c(r, s)$$

where the total cost of the service $c(r, s)$ is a function of the interest rate and agent's savings.

In the market where a monopolistic provider can perfectly observe the agent's characteristics and tailor the contract terms accordingly, the optimal (profit-maximising) savings contract solves the following problem:

$$\max_{r, f} \pi = f - c(r, s), \text{ s.t.:}$$

1. $s = s(\beta, r)$

¹³This proportion equals 59% in the US, 54% in Canada, 100% in the UK, 97% in Australia, 61% in Japan, and 100% in the Netherlands.

¹⁴The model abstracts from financial risk and treats the interest rate as deterministic, but the cited studies find evidence for a significant impact of factors typically associated with higher expected returns, such as the portfolio share of stocks or the 'quality and complexity' of a plan, on the investment costs.

$$2. \hat{V}_0 - f \geq \underline{u}$$

provided that the above results in non-negative profits. The first constraint says that a provider correctly forecasts agent's future savings behaviour given the parameters of the contract.¹⁵ The second constraint is a version of the 'individual rationality constraint', which says that in period 0 the agent's *perceived* utility from signing a contract P cannot be lower than the utility of his outside option. Crucially, note that while the firm's costs of service are a function of β , the agent's valuation of the contract is a function of $\hat{\beta}$.¹⁶

The solution to the contract design problem is derived under Assumption 1, which states that conditional on agent's savings behaviour, the total cost function is convex in the offered rate of return.¹⁷

Assumption 1 *The cost function $c(r, s(\beta, r))$ is convex in r .*

In what follows, the efficiency criterion is given by a social surplus function defined as an equally-weighted sum of utility attained by an agent (as measured by U_0) and firm's profits. Under the assumption of quasi-linearity, this measure of efficiency is a function of the interest rate only. Moreover, it coincides with an objective of a time-consistent individual who has access to firm's technology of converting savings

¹⁵It is standard in the exploitative contracting literature to assume that the firm is able to predict individual's behaviour more accurately than the individual himself. This is primarily due to the firm's superior knowledge of the market and its customers, achieved through expertise, reliance on statistical data, and experience in market interactions (Spiegler 2014).

¹⁶The above formulation of a problem of contract design embeds a couple of important implicit assumptions. First, both the firm and an agent commit to the contract terms. Second, while an agent might hold incorrect beliefs regarding his own future savings behaviour, he understands the contract terms perfectly. Third, there is no uncertainty regarding the rate of return on agent's savings. The second and the third assumption are relaxed in one of the extensions of the model, which does not change the main conclusions of this section.

¹⁷A more specific form of the firm's cost function will be presented in Chapter 3.

into future wealth.

Proposition 1 characterises the main qualitative characteristics of the savings contracts offered in equilibrium, denoted $P^* = (r^*, f^*)$.

Proposition 1 *In a monopolistic market with perfect observability:*

1. *For any r , the fee f^* is increasing in $\hat{\beta}$. For any $\hat{\beta}$, the fee f^* is increasing in r .*
2. *Sophisticates obtain efficient contracts, conditional on β . Due to the monopolistic power of the provider, sophisticated agents obtain utility of \underline{u} .*
3. *Naifs obtain inefficient contracts. The direction of the exploitative distortion is given by:*

$$\frac{dr^*}{d\hat{\beta}} \begin{cases} > 0 & \text{for } \theta < 1, \\ = 0 & \text{for } \theta = 1, \\ < 0 & \text{for } \theta > 1 \end{cases}$$

4. *The agent's utility as well as efficiency are decreasing in the degree of his naiveté, while the firm's profits are increasing in the degree of naiveté.*

The derivation is relegated to the appendix. Following from the second constraint in the optimisation problem, the firm may charge different agent-types different fees for the same rate of return. More specifically, since naïve agents overestimate their future saving, they are willing to accept higher fees for any given interest rate. In addition, the willingness to pay for a savings contract is increasing in the offered rate

of return.

The main efficiency implications of Proposition 1 arise from the fact that the equilibrium contracts maximise the *perceived* consumer-firm surplus. In case of sophisticated individuals, $\hat{\beta} = \beta$ and the perceived surplus coincides exactly with the actual surplus. Then, the equilibrium contracts are efficient. Under the assumption of monopoly, sophisticated agents obtain utility equal to the utility of their outside option, because the firm is able to charge a fee which extracts the entire surplus.

On the other hand, the provider exploits the agent's naiveté about the present bias by offering contract terms that appear attractive provided that an individual saves a lot. The objective of a profit-maximising firm is to exacerbate the agent's forecasting error regarding his future saving and thus his willingness to pay for the contract, relative to the contract's actual worth. This is achieved by distorting the interest rate downwards for $\theta > 1$ when the income effect dominates, and upwards for $\theta < 1$ when the substitution effect dominates, because these distortions raise the naïve agent's forecasted savings. Clearly, the actual savings increase as well, but not as strongly due to the impact of the present bias.

Which of these two cases appears empirically more relevant? Although the CRRA formulation of a utility function is one of the most commonly used in the life-cycle literature, there seems to be no consensus regarding the 'right' size of the CRRA parameter. This is to be expected, as under the parsimonious CRRA formulation, a single parameter governs the elasticity of intertemporal substitution as well as the risk aversion. Nonetheless, most available point estimates indicate $\theta > 1$, while many calibrated life-cycle models assume θ to exceed 2. Thus in relation to the qualitative predictions of the model, the prevalence of inefficiently cheap savings products in the market may be interpreted as a primary policy concern.¹⁸

¹⁸These point estimates regard an average or 'representative' household, but one can imagine that the entire population has economic agents characterised by different magnitudes of the CRRA

The above discussion also highlights the importance of interactions between ‘classical’ (θ) and ‘behavioural’ preference parameters ($\hat{\beta}$) in generating predictions. This issue has thus far received little attention from the literature (Kőszegi 2014). In most cases, comparative statics regarding outcomes obtained by non-classical agents are limited to analysing an isolated impact of a particular behavioural characteristic. Proposition 1 suggests that in certain settings, meaningful interactions with other parameters may also arise. In comparison, while in their seminal paper DellaVigna and Malmendier (2004) already noted that the direction of exploitative distortions to naifs’ contracts is equivocal, their setting did not provide an intuitive condition or a mechanism that would determine it.

Due to this exploitative contract design, naïve agents obtain utility lower than \underline{u} . The resulting loss of consumer welfare is unambiguously increasing in the degree of naiveté and arises from two distinct sources. Due to the ‘distributional effect’, naifs tend to overpay for any given contract terms.¹⁹ Additionally, their contracts are distorted away from the first-best by the provider, which results in the ‘efficiency effect’.²⁰ Finally, the provider’s profits are increasing in the agent’s naiveté.

2.2.1 Endogenous contracts and consumption paths

The above discussion focuses on the efficiency of outcomes and consumer welfare obtained by different agent-types. However, it is also of interest to consider the con-

parameter. For a summary of the existing empirical evidence see Attanasio and Weber (2010). The earliest attempts to estimate the size of the CRRA parameter are outlined in Deaton (1992) and Romer (2006). Examples of calibration exercises that assume $\theta > 2$ are Angeletos et al. (2001), Scholz et al. (2006) and Winter et al. (2012). Moreover, some authors perform sensitivity analyses by assuming θ to lie between 0.5 and 5.

¹⁹As will be shown, greater degrees of competition weaken the negative distributional effect. Similarly for a case when the valuation of agent’s outside option is determined endogenously.

²⁰Such distinction was first noted by DellaVigna and Malmendier (2004). Moreover, note that consistently with an established result, agent’s present bias reduces efficiency and diminishes consumer welfare only when an agent is naïve about it (e.g. DellaVigna, Malmendier 2004; Gabaix, Laibson 2006; Heidhues, Kőszegi 2010).

sumption paths elicited by endogenous contracts.

Corollary 1 *Naifs over-save relative to sophisticates independent of θ . However, they do not necessarily over-accumulate pension wealth.*

It follows directly from Proposition 1 that naïve present-biased agents over-save relative to sophisticated present-biased agents as a result of the exploitative contract design, irrespective of the size of the CRRA parameter θ . Importantly, this does not necessarily imply that naifs over-accumulate pension wealth. Under an arguably more relevant case of $\theta > 1$, naïve agents' higher saving may be offset by an inefficiently low rate of return. As a result, exploitative contracts would induce naifs to under-consume at both stages of the model.²¹

As a side remark, consider a potential intervention of educating individuals, turning all naifs into sophisticates. The above implies that such an intervention would unambiguously lead to a decrease in aggregate saving, even though consumer welfare and efficiency would improve.

2.2.2 A relevant empirical pattern - The design of default options in workplace pensions

For the more relevant case of $\theta > 1$, the prediction of the model may at first seem counter-intuitive. Naïve agents, who overestimate how much they are going to save, are offered inefficiently cheap (low-yield, low-fee) contracts. These exploitative contracts are designed to magnify the difference between their forecasted and actual savings behaviour. There is, however, a salient empirical pattern that is related to

²¹That is true even under a model that ignores the impact of fees paid to the provider on wealth accumulation.

this implication of the model.

Consider automatic enrolment into workplace pension schemes that has been mandated in Italy, New Zealand, Turkey and the UK, and is encouraged by legislation in Canada and in the US (OECD 2017). Under automatic enrolment, eligible workers are required to actively opt-out of a workplace pension scheme if they do not wish to participate, rather than to actively opt-in. This change of the default option, without affecting the economic incentives, has been shown by multiple studies to have a dramatic impact on an array of outcomes, from participation in the scheme to contribution rates, asset allocations, and resulting variation in wealth accumulation across workers (see e.g. Benartzi, Thaler 2004, 2007; Beshears et al. 2009; Choi et al. 2004; Madrian, Shea 2001). Indeed, the prevalence of the default option effects in the domain of retirement saving remains one of the most powerful results in empirical behavioural economics.

Various demand-side explanations have been put forward to explain why individuals become so strongly anchored in the default savings options. These include transaction costs, inattention, procrastination, and financial illiteracy.²² However, I am not aware of a study that would account for the impact of the design of the default option itself on the strength of the default option effect.²³ As noted in the literature, the typical default savings option combines moderate contribution rates with a conservative asset allocation yielding low returns, such as a money market fund. Whether by exploitative design or due to other factors (e.g. employer's costs of provision of a pension plan), the present model suggests that such low-yielding

²²For example, models with present-biased preferences can predict low switching rates from inferior default options despite low transaction costs (O'Donoghue, Rabin 1999a, 2001; Choi et al. 2003).

²³Choi et al. (2003) and Carroll et al. (2009) consider a socially optimal choice of the default savings rate when workers have heterogeneous preferences over savings rates. However, these analyses abstract from other characteristics of the default option, such as asset allocation and expected returns, as well as from the incentives faced by the providers.

default allocations only exacerbate the difference between individual's expected and actual savings behaviour, thereby strengthening the default option effect.²⁴ Put differently, the existing evidence on the default option effect should be interpreted as conditional on particular parameters of the default.²⁵

2.2.3 Alternative modelling assumptions

The baseline formulation of the model makes several modelling assumptions that improve the clarity of exposition. While a separate section is devoted to extensions of the model, the following discusses several important assumptions underpinning all formulations of the model. These are assumptions about the contract space, agent's outside option and uncertainty.

Contingent contract terms. In the baseline model, the interest rate r and the fee f are unconditional, i.e. independent of the agent's actions (savings). Note that in the above setting that is a genuine constraint. For example, under a richer contract space, the sophisticated agents would opt for commitment contracts that counteract the impact of their future present bias. On the other hand, in the case of naifs, the unconditional contracts reduce the extent of exploitation and limit the related welfare losses. If the contract parameters were contingent on future actions, naïve agents would overvalue savings contracts to an even greater extent.²⁶

²⁴Although it may appear that there is no ambiguity regarding future saving levels once a contribution rate is known, a naïve individual may nonetheless (incorrectly) expect to revise his saving upwards by opting for a higher contribution rate in the future.

²⁵In reality, employers play a role of an intermediary, who chooses a particular pension provider on behalf of their employees. Under a stylised view, an employer would select a pension arrangement that maximises employees' utility subject to a restriction on fees, for example due to the shared costs of provision. Analogously to the baseline model, an employer may well aim to maximise employees' perceived utility from a savings contract thereby maximising the perceived, rather than the realised, value of the benefit. Then, the impact of employer's intermediation would be equivalent to the effect of introducing a ceiling on fees, which is discussed below. A potential discrepancy between employer's and employees' preferences and/or beliefs posts a separate research question.

²⁶A general result from the behavioural industrial organisation literature (e.g. DellaVigna, Mal-mendier 2004; Spiegel 2014) states that under unconstrained contracting, sophisticated agents

Endogenous outside options. The baseline model assumes exogenous and type-independent valuation of the outside option \underline{u} . Such an assumption is justified when the utility from selecting the outside option does not depend on agent's own actions, for example when the outside option corresponds to reliance on a mandatory system of public pension benefits. However, one can also imagine that the valuation of an outside option is a function of the agent's future actions, for example when the outside option involves saving into an account offered by an external provider (e.g. a bank). In contrast to the baseline model, suppose that the agent's outside option involves saving into a costless savings account offering a rate of return \underline{r} . Then, the valuation of the outside option becomes endogenous, $\underline{u} = V_0(s(\hat{\beta}, \underline{r}), \underline{r})$. Although a fuller treatment of this case is relegated to the appendix, it is evident that naïve agents overestimate the value of their outside option, which affects their willingness to pay for a savings contract. While this mechanism curbs the negative distributional effect, it has no impact on the efficiency properties of the savings contracts offered in equilibrium.²⁷

Uncertainty. Uncertainty can be introduced into the model via several alternative channels. First, imagine that realisation of the present bias parameter $\beta < 1$ is a chance event happening with probability $p \in [0, 1]$. Otherwise, there is no present bias (i.e. $\beta = 1$ with probability $(1 - p)$). If an agent has a correct prior about p ,

select perfect commitment contracts that induce them to act as if they were time-consistent. Furthermore, the literature shows that restricting attention to two-part tariffs generates the same outcome. However, this does not extend to the above setup, in which the two-attributed equilibrium contracts $P^* = (r^*, f^*)$ for sophisticated agents do not provide perfect commitment. That is because the action that is desired from a period-0 perspective is itself a function of the contract parameter r . In other words, there is no parameter of the contract that would play the role of a per-unit price p from the standard models. In those models, when the desired action ('quantity purchased') is independent of p , the price can be selected so as to incentivise the agent's future self to take that action.

²⁷More specifically, the negative distributional effect experienced by naifs is curbed for $\theta < 1$ and even becomes positive for $\theta > 1$. Under perfect competition, however, there is no role for the distributional effect, and naifs are unambiguously worse off than sophisticates due to the prevalence of the adverse efficiency effect. The case of competition is examined in section 2.4.

but still underestimates the intensity of the present bias (i.e. $\hat{\beta} \geq \beta$), such a model is characterised by ‘magnitude naiveté’, but not ‘frequency naiveté’. Then, the qualitative properties of the exploitative contracts carry over from the baseline model, but the quantitative impact is now weighted down by the corresponding probability p . Magnitude naiveté combined with frequency naiveté, where $\hat{p} \leq p$, would only strengthen the exploitative motive. Second, uncertainty could relate to the realised rate of return on the agent’s savings. If randomness was due to ‘nature’, that is the provider bears a cost of generating the rate of return r while the realised rate of return is $r + \epsilon$ where ϵ denotes a random component, the qualitative properties of the exploitative savings contracts would carry over from the deterministic setting. A case in which uncertainty about future returns reflects a direct choice of the provider is covered by the extension ‘Financial (il)literacy’.

2.3 Policy interventions

Numerous reports highlight the need for appropriate regulation of markets for pension products (e.g. OECD 2016, 2017; Office of Fair Trading 2014). This section briefly examines the effects of three widespread policy interventions: imposing a ceiling on fees, regulating competition, and introducing a minimum savings requirement. Note that timing is crucial for predictions. It is assumed that the firm offers a savings contract after the government has taken its respective action. In other words, the firm is allowed to respond to the regulation.

The following examples highlight the challenges to policy making in this domain. A lack of a simple and effective policy remedy is due to several factors. First, the timing assumption. Second, the potential dependence of a welfare improving policy on the size of the CRRA parameter θ , which is not only difficult to estimate, but

most likely varies across individuals. Third, the fact that under any market conditions naifs select the contracts that maximise their *perceived*, rather than actual welfare.

A fuller treatment is presented in the appendix.

2.3.1 Ceiling on fees

The most popular regulations restrict types and amounts of fees that may be charged by pension providers and are observed across many OECD countries (Dobronogov, Murthi 2005; Tapia, Yermo 2008). Consider imposing a ceiling on fees, denoted \bar{f} . Then, the optimisation problem of the firm is subject to an additional constraint:

$$\max_{r,f} \pi = f - c(r, s), \text{ s.t.:}$$

1. $s = s(\beta, r)$
2. $\hat{V}_0 - f \geq \underline{u}$
3. $f \leq \bar{f}$

Lemma 2 characterises the impact of a ceiling on savings contracts offered in equilibrium.

Lemma 2 *Under an effective ceiling on fees:*

- *All interest rates are revised downwards.*
- *The efficiency of sophisticated agent's contracts declines, but consumer welfare is preserved.*

- *Efficiency and consumer welfare attained by naïve agents improve for $\theta < 1$, but decline for $\theta > 1$.*
- *Firm's profits decrease.*

An effective ceiling does not only mechanically reduce the transfer f between an agent and the firm. It also distorts the equilibrium interest rates. Note that under the baseline model, both fees charged by the firm as well as firm's costs are increasing in the interest rate. Thus after an imposition of an effective ceiling, the profit-maximising interest rates are revised downwards. For sophisticated agents, if they happen to be affected by the policy, such a shift implies no change in consumer welfare, but it strictly decreases efficiency as outcomes are forced away from the first-best. For naïve agents, on the other hand, the impact on both welfare and efficiency crucially depends on the size of the CRRA parameter θ . In fact, the policy has a detrimental effect for the case of $\theta > 1$, when the exploitative contracts are 'inefficiently cheap' to start with. If a policymaker indeed suspects the savings contracts to be inefficiently cheap, a regulation 'opposite' to the ceiling on fees is desirable, such as enforcing return guarantees.²⁸ Independent of the scenario, however, firm's profits shrink. Thus another undesired effect of such a policy is the possibility that the provider would not be able to generate non-negative profits after an imposition of too strict a ceiling.

²⁸In practice, enforcing guarantees of generous rates of return may well be prohibitively costly due to financial risk inherent in most investment strategies. Furthermore, such guarantees are not easily implementable due to various administrative and institutional barriers (OECD 2011).

2.3.2 Regulating competition

There are several ways in which policy may affect the degree of competition in the market, for example by regulating entry or enabling individuals to switch providers (see also the extensions that introduce perfect and imperfect competition more formally). For instance, Tapia and Yermo (2008) discuss how differences in the regulatory framework across OECD countries influence competition and pricing of pension products. Apart from playing a role of a regulator, government is also an additional provider of pension benefits operating through a social security system. Recent pension reforms in most OECD countries have substantially reduced the value of state-funded benefits (OECD 2016). Intuitively, the generosity of public benefits is likely to affect the agent's willingness to sign a private pension contract and thus influence the degree of price competition as well. Such interventions can be captured by the model simply as an exogenous change in \underline{u} . The following is a direct implication of the proof of Proposition 1.

Corollary 2 *An increase in \underline{u} lowers f^* , but has no effect on r^* .*

When an agent has access to a more attractive outside option, the firm charges lower fees for every interest rate, which improves consumer welfare at the expense of firm's profits. However, there is no impact on the efficiency properties of equilibrium contracts. That is because the exploitative savings contracts maximise the *perceived* consumer surplus. In other words, an intervention that improves the agent's outside option will curb the negative distributional effect, but will have no impact on the efficiency effect. Conversely, a decrease in \underline{u} will result in higher fees, redistributing the wealth from the agent to the firm.

2.3.3 Minimum savings requirement

Government might also impose a lower bound on levels of saving by making private pension contributions compulsory and specifying the minimum contribution rate.²⁹ Examples of such policies come from Australia and the UK, where minimum contributions into private pension schemes have been set to ensure adequate saving. Suppose that the government introduces a minimum savings requirement \underline{s} . Then, the optimal contract offer solves the following:

$$\max_{r,f} \pi = f - c(r, s), \text{ s.t.:}$$

1. $s = \max \{s(\beta, r), \underline{s}\}$
2. $\hat{V}_0 - f \geq \underline{u}$

where $\hat{s} = \max \{s(\hat{\beta}, r), \underline{s}\}$. Note an interesting possibility that the requirement is in fact binding, while an agent does not realise this (i.e. $s = \underline{s}$ while $\hat{s} > \underline{s}$). Lemma 3 describes the impact of the requirement on the equilibrium contract terms.

Lemma 3 *The impact of a minimum savings requirement on interest rates offered in market equilibrium is ambiguous in general. In the neighbourhood of $\hat{\beta} = 1$, however, the minimum savings requirement (weakly) exacerbates the exploitative contract characteristics, which reduces efficiency and consumer welfare.*

By offering varying interest rates, the provider effectively determines whether the minimum savings requirement is binding and whether an agent realises this, as both

²⁹A theoretical argument in favour of this policy is provided by Amador et al. (2006) who study optimal commitment contracts in presence of random shocks. They show that the optimal commitment devices always feature a restriction resembling the minimum savings requirement.

actual and predicted savings are a function of r . Moreover, either contract design strategy could be profit-maximising, depending on parameter values. For example, a binding minimum savings requirement that is correctly acknowledged by an agent eliminates firm's profits due to naiveté, but at the same time it provides commitment which increases the agent's valuation of a savings contract. Therefore the impact a minimum savings requirement on the contract design is ambiguous.

However, the model yields a more specific prediction for the case of (nearly) complete naiveté, i.e. $\hat{\beta} \approx 1$. Then, the minimum savings requirement results in offers of (weakly) higher interest rates for $\theta < 1$ and (weakly) lower interest rates for $\theta > 1$, which only exacerbates the original exploitative contract features. Recall that under the baseline model, the exploitative contracts unambiguously induced 'over-saving' by naifs. Consequently, enforcing even higher levels of saving, without affecting the beliefs guiding the valuation of a savings contract, cannot lead to improved market outcomes.³⁰

2.4 Extensions

The baseline model provides a clear illustration of the main mechanism behind the exploitative design of saving contracts. These key results carry over, in qualitative sense, to increasingly competitive environments, to a case when the provider charges variable fees, and to a case when the agent may be financially unsophisticated and therefore misinterpret the contract terms. I also discuss how imperfect observability of the individual characteristics affects the optimal (profit-maximising) contract de-

³⁰To illustrate the importance of the timing assumption, note that if the government set the minimum savings requirement after the provider has offered the interest rate, it would optimally enforce all agents to save as much as if they were time-consistent by setting $\underline{s} = s(1, r)$. Then, the forecasting errors and welfare losses due to agents' naiveté would not arise, and outcomes would be equalised across different agent-types. However, this does not necessarily imply greater efficiency.

sign. A fuller treatment of all extensions is provided in the appendix.

Perfect competition. Under perfect competition, many homogeneous providers freely enter the market, which increases the (endogenous) value of the agent’s outside option until each firm’s profits are zero at the optimum. Because the entire consumer surplus is distributed to the individual, all agent-types are better off under perfect competition, relative to the baseline case of monopoly, while the firm’s profits decline. What is more, competition does away with the negative distributional effect, due to which naïve agents were overpaying for their contract terms. As implied by the zero-profit condition, the competitive fees cover the actual cost of the service and are thus independent of the agent’s valuation of a contract. However, the inefficient distortions to rates of return offered to naïve individuals persist despite competition, and so does the negative efficiency effect. That is again because the exploitative contract terms maximise the *perceived* consumer surplus. As a result, naifs remain worse off than their sophisticated counterparts.³¹

Imperfect competition. In the appendix, I consider a Hotelling model of competition between two homogeneous, spatially separated firms. A pure-strategy equilibrium of the model is symmetric, involves both firms offering the same interest rate as under monopoly and under perfect competition cases, and implies a common fee that changes monotonically between the perfectly competitive and monopolistic levels according to a single parameter of ‘distance aversion’. In a context of a market for financial products, this parameter is more naturally interpreted as capturing the agents’ tendency to prefer a ‘default’ provider.³²

³¹As a side remark, note that in reality greater competition in a market for pension products does not necessarily reduce the fees, as providers may incur additional marketing costs (Tapia, Yermo 2008).

³²This modelling choice follows the approach in Heidhues, Kőszegi (2010). There are alternative models which, depending on a parameter governing the degree of competition or the severity of information (search) frictions, generate a distribution of prices for a homogeneous good that ranges from the perfectly competitive to the monopolistic price. These include Butters (1977), Burdett and Judd (1983), and Stahl (1989). Advantage of these models is perhaps a more natural interpretation

Variable fees. Variable fees, proportional to total assets, contributions, or interest earnings, are commonly observed in the pension industry (Dobronogov, Murthi 2005; Tapia, Yermo 2008). What is important, variable fees make the provider’s incentives more aligned with the agent’s preferences. For example, with variable fees proportional to the agent’s accumulated wealth, the firm has a greater incentive to encourage high levels of saving and to provide high rates of return.³³ Under the baseline model, on the other hand, the firm would prefer to charge as high a fee as possible while providing a service of little actual benefit. It is thus of interest to extend the model to allow for a richer fee structure. Indeed, variable fees proportional either to the agent’s savings or to the accumulated wealth affect the optimal (profit-maximising) contract design in the expected way. However, the exploitative distortions to the offered interest rates are qualitatively the same as under the model with flat-rate fees only.

Financial (il)literacy. A growing body of empirical and theoretical work highlights the important role of financial literacy in economic decision-making and documents a pervasive lack thereof (Lusardi, Mitchell 2014; Lusardi et al. 2017). Indeed, low levels of financial literacy combined with increasingly consequential and complex financial decisions that individuals face appear to have adverse effects on a range of outcomes, in particular on wealth accumulation (Jappelli 2010; Lusardi, Mitchell 2007; Van Rooij et al. 2012). What is more, some authors have argued that certain financial products, such as credit cards or retail structured products, exhibit exploitative characteristics designed to take advantage of consumers’ limited financial sophistication (Heidhues, Kőszegi 2010; Célérier, Vallée 2017). The appendix out-

of key parameters. However, their disadvantages, relative to the simple Hotelling formulation, are the introduction of multiple free parameters, necessary reliance on mixed-strategy equilibria, and potential multiplicity of equilibria.

³³I am thankful to one of the managers at a big pension provider in the UK for a helpful conversation and for pointing this out.

lines a simple version of the model that allows for misinterpretation of the contract terms by the individuals. More specifically, the firm advertises both a ‘headline’ and an ‘actual’ fee (or a ‘headline’ and an ‘actual’ interest rate), and a financially naïve agent overestimates the importance of those ‘headline’ contract parameters. Although such financial naiveté affects the contract design in the predictable way (i.e. the ‘headline’ contract parameters appear more attractive), the qualitative properties of the exploitative distortions due to individual’s naiveté about the present bias carry over from the baseline model.

Imperfect observability with homogeneous beliefs. An important extension relaxes the assumption of perfect observability of the individual characteristics by the firm. Consider a case when the firm serves two indistinguishable types of agents with homogeneous beliefs about their present bias, but with different actual magnitudes of the bias. Without loss of generality, suppose that a fraction $\lambda \in [0, 1]$ of agents is completely naïve ($\beta < 1$ and $\hat{\beta} = 1$) and that a fraction $1 - \lambda$ is time-consistent ($\beta = \hat{\beta} = 1$).³⁴ As the two types share their beliefs ($\hat{\beta} = 1$), the firm cannot design screening contracts. That is because whichever contract is accepted (preferred) by a time-consistent agent is also accepted (preferred) by a naïf. Consequently, such a market is characterised by pooling and no exclusion. The firm’s optimal (profit-maximising) contract offer solves:

$$\max_{r,f} \mathbb{E}[\pi] = \mathbb{E}[f - c(r, s)], \text{ s.t.:}$$

1. $s = s(\beta, r)$ with probability λ ; $s = s(1, r)$ with probability $(1 - \lambda)$
2. $V_0(s(1, r), r) - f \geq \underline{u}$

³⁴The same qualitative results can be obtained for a more general case of two agent-types characterised by the same $\hat{\beta} \leq 1$, but various $\beta^L < \beta^H = \hat{\beta}$.

As a result of pooling, contracts for sophisticated, time-consistent individuals are no longer efficient. However, due to their correct valuation of the contract offer, the consumer welfare of sophisticates is preserved. Note that the loss of efficiency of sophisticates' contracts is not necessarily compensated for by an improvement in the efficiency of naifs' contracts. Under pooling, naïve present-biased individuals may either obtain more efficient savings contracts (and be better off) or obtain even less efficient contracts (and be worse off), depending on the direction of the original exploitative distortion and a cross-partial derivative of the firm's cost function. For example, if it is increasingly expensive to offer a high rate of return for high savers (e.g. due to limits to arbitrage), a pooling contract will be 'cheaper' relative to the exploitative contract offered to naifs in isolation. This counteracts the exploitative distortion for $\theta < 1$, when the exploitative contract is inefficiently 'expensive', but only worsens it for $\theta > 1$.³⁵ ³⁶

Imperfect observability with heterogeneous beliefs. Alternatively, assume that a market for savings products is populated by two indistinguishable types of agents characterised by the same magnitude of their present bias, but different beliefs about the bias. Without loss of generality, consider a population consisting of a fraction $\lambda \in [0, 1]$ of naïve present-biased agents ($\beta < 1$ and $\hat{\beta} = 1$) and a fraction $1 -$

³⁵More precisely, when $\frac{d^2 c(r,s)}{ds dr} < 0$ (e.g. due to scale effects or a greater bargaining power of a pension fund with a large portfolio), naifs are offered higher interest rates than in isolation, which counteracts the exploitative distortion for $\theta > 1$, but only exacerbates it for $\theta < 1$. Sophisticates are offered lower interest rates than in isolation, which unambiguously diminishes efficiency while preserving consumer welfare. When $\frac{d^2 c(r,s)}{ds dr} > 0$ (e.g. due to limits to arbitrage or liquidity risk), naifs are offered lower interest rates than in isolation, which counteracts the exploitative distortion for $\theta < 1$, but only exacerbates it for $\theta > 1$. Sophisticates are offered higher interest rates than in isolation, which unambiguously diminishes efficiency while preserving consumer welfare.

³⁶For completeness, note that the potential concern about a lack of equilibrium under competition and imperfect observability, as in Rothschild and Stiglitz (1976), does not apply in this case. With homogeneous beliefs $\hat{\beta}$, the two agent-types have identical preferences over contracts. In other words, there is no profitable deviation from the equilibrium pooling contract that attracts just one of the types.

λ of sophisticated present-biased agents ($\beta = \hat{\beta} < 1$).³⁷ Distinctly from the preceding case with homogeneous beliefs, a monopolistic provider may, but does not need to, differentiate the contract offers in order to separate the two agent types. For illustration, consider the optimal design of a screening contract. In addition to the constraints introduced earlier, such a contract must be incentive compatible in the sense that neither of the agent-types has an incentive to mimic the other type. The firm's problem becomes:

$$\max_{r^N, r^S, f^N, f^S} \mathbb{E}[\pi] = \lambda\{f^N - c(r^N, s^N)\} + (1 - \lambda)\{f^S - c(r^S, s^S)\}, \text{ s.t.:}$$

1. $s^N = s(\beta, r^N)$
2. $s^S = s(\beta, r^S)$
3. $V_0(s(1, r^N), r^N) - f^N \geq \underline{u}$
4. $V_0(s(\beta, r^S), r^S) - f^S \geq \underline{u}$
5. $V_0(s(1, r^N), r^N) - f^N \geq V_0(s(1, r^S), r^S) - f^S$
6. $V_0(s(\beta, r^S), r^S) - f^S \geq V_0(s(\beta, r^N), r^N) - f^N$

where superscripts N and S refer to naifs and sophisticates respectively. While constraints 1-4 are the same as under the baseline problem, constraints 5 and 6 assure that no agent-type prefers a contract designed for the other type ('no mimicking'). In the appendix I demonstrate that while constraint 5 is binding, constraint 6 is slack. Intuitively, only naifs may have an incentive to prefer the contract offered to sophisticated individuals as sophisticates do not overpay for their contracts. Moreover, at the optimum, the rate of return for naïve agents (r^N) is the same as under

³⁷The same qualitative results can be obtained for a more general case of two agent-types characterised by the same $\beta \leq 1$, but various $\hat{\beta}^H > \hat{\beta}^L \geq \beta$.

isolation, while the rate of return for sophisticates (r^S) is distorted away from the first-best. Naifs do pay lower fees than under perfect observability, however, in order to satisfy the no-mimicking constraint 5.

In addition, the firm may also design its offer in order to serve naïve individuals only (exclusion of sophisticates) or serve both agent-types without separating them (a pooling contract). Depending on the parametrisation of the model (in particular, the population composition as given by λ and the magnitude of the present bias β), either of the three contract design strategies may maximise profits. What is important, however, is the fact that independently of the contract design strategy, naifs are (weakly) better off relative to perfect observability, while sophisticates obtain (weakly) less efficient savings contracts but retain their utility. Thus the impact of imperfect observability on welfare attained by naïve individuals crucially depends on the particular population composition. While naifs can be made worse off by pooling them with time-consistent agents, they are unambiguously better off as members of the uniformly present-biased population.³⁸

2.5 Conclusion

Chapters 2 and 3 study the interaction between a present-biased individual and a private pension provider. A simple baseline model indicates that ‘sophisticates’, that is agents who are fully aware of their present bias, receive efficient contract offers, while ‘naifs’, who are at least partially unaware of their present bias, receive ex-

³⁸As a final remark, note that for this population composition, competitive equilibrium consists of two zero-profit contracts, one of which maximises sophisticates’ perceived utility and another maximises naifs’ perceived utility. These are the same contracts as offered in isolated markets under perfect competition. Again, the potential issue of a lack of competitive equilibrium does not apply. That is because the difference in agents’ $\hat{\beta}$ should be interpreted as difference in ‘tastes’ rather than ‘risks’ (costs) to the providers. In other words, there is no adverse selection under the zero-profit contracts.

ploitative contract offers. The agent's naiveté about the present bias is exploited by offering contract terms that appear attractive as long as the individual saves a lot. This is achieved by distorting the interest rate downwards when the income effect of an interest rate change dominates in the agent's utility function, and upwards when the substitution effect dominates. Because the central properties of the exploitative savings contracts depend on the curvature of the utility function, which is not only difficult to estimate but is also likely to vary across individuals, certain well-intended policy interventions, such as ceilings on fees or minimum savings requirements, might decrease both efficiency and consumer welfare. Moreover, since under any market conditions the exploitative contracts maximise the perceived consumer-firm surplus, the issue of inefficiency is more difficult to resolve than concerns associated with the distribution of wealth.

Regarding future research directions, the model might be extended in a number of ways in order to provide additional insight and to address remaining policy questions. First, what if the interactions between a firm and a consumer are repeated, and an individual is allowed to either renegotiate contract terms or sign multiple contracts? Then, the firm's optimal strategy should take into account both profits from a single interaction and gains from extending the existing relationship with an individual. In such a dynamic setting, what is the optimal timing of a contract offer? A second interesting extension might include the possibility to learn about the present bias. Does exploitation survive in the long run? Third, regarding the increasingly important workplace pension arrangements, what is the role of an intermediation by an employer if her preferences do not necessarily coincide with those of her workers? All these questions call for further research.

Appendix

Quasi-linearity of the utility function

For several reasons, the utility function assumed in the baseline model is linear in transfer f between an agent and a firm. First, that makes the interest rate r the single parameter of the contract that affects the agent's intertemporal trade-off. In a context of life-cycle saving, this assumption does not seem overly restrictive. As a result, the interest rate is chosen by the provider in order to maximise the *perceived* consumer surplus, while the fee is used to extract the surplus from the agent. Second, the assumption of quasi-linearity results in a clear efficiency criterion, which is not affected by the size of a transfer between the agent and the firm ('money changing hands'). Put differently, the resulting efficiency criterion would be an objective function of an agent who has access to the firm's technology of converting savings into future wealth, akin a Robinson economy. Third, the analysis is simplified, which is achieved without significant loss of insight as long as the impact of the fee on the intertemporal trade-off remains 'small' (see below). Lastly, such an approach appears to be standard in the behavioural contracting literature (e.g. Eliaz, Spiegel 2006; Gabaix Laibson 2006; Grubb 2009).

For illustration, consider an alternative formulation of the model which relaxes the assumption of quasi-linearity:

$$U_1 = u(c_1) + \beta u(c_2),$$

where $c_1 = Y - s - f$ and $c_2 = (1 + r)s$. Agent's saving $s = s(\beta, r, f)$ that maximises U_1 , is affected by the fee f in the same way as by any exogenous shock to the level of income Y :

$$s(\beta, r, f) = \frac{1}{1 + \beta^{\frac{-1}{\theta}} (1+r)^{\frac{\theta-1}{\theta}}} \times [Y - f]$$

Thus as long as $f \ll Y$, the difference $s(\beta, r, f) - s(\beta, r, 0)$ will be quantitatively ‘small’, or $\frac{ds(\beta, r, f)}{df} \approx 0$.³⁹ In that case, the solution to the firm’s problem of contract design would be characterised by the same qualitative properties as under the baseline model. That is because the interest rate r is selected by the provider to maximise the agent’s perceived surplus, while the fee f extracts the surplus.

In a model in which the provider charges fees f_1 and f_2 in periods 1 and 2 respectively, and $f_1 \neq f_2$ is allowed, one can imagine a contract design under which the fee schedule effectively acts as a commitment device (e.g. $f_1 > 0$ and $f_2 < 0$). However, modelling the provision of commitment devices is not subject of the present work. Neither do fees for financial services typically play that role.

Lemma 1

A consumption schedule (c_1, c_2) , where $c_1 = Y - s$ and $c_2 = (1 + r)s$, that maximises U_0 satisfies the following first order condition:

$$u'(Y - s) = (1 + r) u'((1 + r)s)$$

The expected saving \hat{s} , on the other hand, maximises U_1 parametrised by $\hat{\beta}$, and thus satisfies the following condition:

$$u'(Y - \hat{s}) = (1 + r) \hat{\beta} u'((1 + r)\hat{s})$$

It follows that naifs overestimate their future saving $\hat{s} \equiv s(\hat{\beta}, r)$ for a concave $u(\cdot)$, i.e. $\frac{d\hat{s}}{d\hat{\beta}} > 0$. Consequently, naifs overvalue any savings contract $P = (r, f)$. The impact of the agent’s beliefs on valuation of a contract is given by:

$$\frac{d\hat{U}_0(P)}{d\hat{\beta}} = \frac{d\hat{V}_0(P)}{d\hat{\beta}} = \underbrace{[-u'(Y - \hat{s}) + (1 + r) u'((1 + r)\hat{s})]}_{\geq 0} \underbrace{\frac{d\hat{s}}{d\hat{\beta}}}_{> 0} \geq 0$$

for $\hat{\beta} \leq 1$. Moreover, the first term is strictly positive for $\hat{\beta} < 1$.

³⁹The same remarks apply to savings behaviour $\hat{s} = \hat{s}(\hat{\beta}, r, f)$ predicted in period 0.

Proposition 1

Recall the firm's problem of optimal contract design in a monopolistic market with perfect observability:

$$\max_{r,f} \pi = f - c(r, s), \text{ s.t.}:$$

1. $s = s(\beta, r)$
2. $\hat{V}_0 - f \geq \underline{u}$

The second constraint also holds as equality since $\frac{d\pi}{df} > 0$. Then, the first statement in Proposition 1 follows directly from Lemma 1 and from the fact that $\frac{d\hat{V}_0}{dr} > 0$ around $\theta = 1$.⁴⁰

Substituting for both constraints reduces the problem to:

$$\max_r V_0(s(\hat{\beta}, r), r) - \underline{u} - c(r, s(\beta, r))$$

Define the efficiency criterion as:

$$\begin{aligned} SS(P) &= U_0(P) + \pi(P) = V_0(s(\beta, r), r) - f + f - c(r, s(\beta, r)) = \\ &= V_0(s(\beta, r), r) - c(r, s(\beta, r)) \end{aligned}$$

An efficient contract has r that satisfies:

$$\frac{dV_0(s(\beta, r), r)}{dr} = \frac{dc(r, s(\beta, r))}{dr}$$

⁴⁰Naturally, $\frac{\partial \hat{V}_0}{\partial r} > 0$ everywhere, but bear in mind that $\frac{d\hat{V}_0}{dr} = \frac{\partial \hat{V}_0}{\partial r} + \frac{\partial \hat{V}_0}{\partial \hat{s}} \frac{d\hat{s}}{dr}$ and the sign of the second term changes around $\theta = 1$.

while under the assumption of quasi-linear preferences, efficiency is invariant to the transfer f between an agent and the firm.

For a sophisticated agent, $\hat{\beta} = \beta$ and thus the firm's optimal contract solves:

$$\max_r V_0(s(\beta, r), r) - \underline{u} - c(r, s(\beta, r))$$

the solution of which coincides with the efficiency criterion. This demonstrates that sophisticated present-biased agents obtain contracts that are efficient, conditional on β .

For a naïve agent, on the other hand, $\hat{\beta} > \beta$ and the optimal contract solves:

$$\max_r V_0(s(\hat{\beta}, r), r) - \underline{u} - c(r, s(\beta, r))$$

or:

$$\frac{d V_0(s(\hat{\beta}, r), r)}{dr} = \frac{d c(r, s(\beta, r))}{dr}$$

which does not coincide with the efficiency criterion as long as

$$\frac{d^2 V_0(s(\hat{\beta}, r), r)}{d\hat{\beta} dr} \neq 0$$

Applying total differentiation to \hat{V}_0 yields:

$$\frac{d V_0(s(\hat{\beta}, r), r)}{dr} = \frac{\partial V_0(s(\hat{\beta}, r), r)}{\partial r} + \frac{\partial V_0(s(\hat{\beta}, r), r)}{\partial s(\hat{\beta}, r)} \frac{d s(\hat{\beta}, r)}{dr}$$

and then:

$$\frac{d^2 V_0(s(\hat{\beta}, r), r)}{d\hat{\beta} dr} = \underbrace{\frac{\partial^2 \hat{V}_0}{\partial \hat{s} \partial r} \frac{d \hat{s}}{d \hat{\beta}}}_{>0} + \underbrace{\frac{\partial^2 \hat{V}_0}{\partial \hat{s}^2} \frac{d \hat{s}}{dr} \frac{d \hat{s}}{d \hat{\beta}}}_{>0} + \underbrace{\frac{\partial \hat{V}_0}{\partial \hat{s}} \frac{d^2 \hat{s}}{d \hat{\beta} dr}}_{\geq 0}$$

In the above, the inequalities follow from the derivation of Lemma 1.

Under the CRRA utility formulation \hat{s} does have a closed form solution, which satisfies the following first order condition:

$$u'(Y - \hat{s}) = (1 + r) \hat{\beta} u'((1 + r)\hat{s})$$

where $u(x) = \frac{x^{1-\theta}-1}{1-\theta}$ and $\theta > 0$. Then:

$$\hat{s} = \frac{1}{1+\hat{\beta}^{-\frac{1}{\theta}}(1+r)^{\frac{\theta-1}{\theta}}} \times Y$$

Differentiation shows that $\frac{d\hat{s}}{d\hat{\beta}} > 0$, while

$$\frac{d\hat{s}}{dr} \begin{cases} > 0 & \text{for } \theta < 1, \\ = 0 & \text{for } \theta = 1 \\ < 0 & \text{for } \theta > 1 \end{cases}$$

in line with the main body discussion regarding relative strength of the substitution and income effects and the size of the CRRA parameter θ .

Taking into account that

$$\hat{V}_0 = \frac{[Y-\hat{s}]^{1-\theta}-1}{1-\theta} + \frac{[(1+r)\hat{s}]^{1-\theta}-1}{1-\theta}$$

the following can be shown using algebraic rearrangements:

$$\frac{\partial^2 \hat{V}_0}{\partial \hat{s} \partial r} \begin{cases} > 0 & \text{for } \theta < 1, \\ = 0 & \text{for } \theta = 1 \\ < 0 & \text{for } \theta > 1 \end{cases}$$

$$\frac{\partial^2 \hat{V}_0}{\partial \hat{s}^2} < 0$$

$$\text{and } \frac{d^2 \hat{s}}{d\hat{\beta} dr} \begin{cases} > 0 & \text{for } \theta < 1, \\ = 0 & \text{for } \theta = 1 \\ < 0 & \text{for } \theta > 1 \end{cases} \quad \text{around } \theta = 1.$$

Therefore:

$$\frac{d^2 V_0(s(\hat{\beta}, r), r)}{d\hat{\beta} dr} = \underbrace{\frac{\partial^2 \hat{V}_0}{\partial \hat{s} \partial r} \frac{d\hat{s}}{d\hat{\beta}}}_{\begin{cases} > 0 & \text{for } \theta < 1 \\ < 0 & \text{for } \theta > 1 \end{cases}} + \underbrace{\frac{\partial^2 \hat{V}_0}{\partial \hat{s}^2} \frac{d\hat{s}}{dr} \frac{d\hat{s}}{d\hat{\beta}}}_{\begin{cases} < 0 & \text{for } \theta < 1 \\ > 0 & \text{for } \theta > 1 \end{cases}} + \underbrace{\frac{\partial \hat{V}_0}{\partial \hat{s}} \frac{d^2 \hat{s}}{d\hat{\beta} dr}}_{\begin{cases} > 0 & \text{for } \theta < 1 \\ < 0 & \text{for } \theta > 1 \end{cases}}$$

Importantly, for every parameter constellation that has $\hat{\beta} \leq 1$,

$$|\frac{\partial^2 \hat{V}_0}{\partial \hat{s} \partial r}| \geq |\frac{\partial^2 \hat{V}_0}{\partial \hat{s}^2} \frac{d\hat{s}}{dr}|$$

Finally:

$$\frac{d^2 V_0(s(\hat{\beta}, r), r)}{d\hat{\beta} dr} \begin{cases} > 0 & \text{for } \theta < 1 \\ = 0 & \text{for } \theta = 1 \\ < 0 & \text{for } \theta > 1 \end{cases}$$

Intuitively, the above determines the direction of changes to the interest rate that exacerbate the overvaluation of a savings contract by a naïve agent $\frac{dV_0(s(\hat{\beta}, r), r)}{d\hat{\beta}}$. This condition combined with Assumption 1 implies the main result regarding the direction of exploitative distortions, or $\frac{dr^*}{d\hat{\beta}}$.

Regarding efficiency, it is evident that while savings contract for sophisticated agents maximise social surplus, the contracts for naïve agents are inefficiently distorted, and the more so the greater the degree of naiveté. Thus social surplus is decreasing in $\hat{\beta}$, holding β fixed.

At the optimum, the utility obtained by an agent is:

$$U_0(P^*) = V_0(s(\beta, r^*), r^*) - f^* = V_0(s(\beta, r^*), r^*) - V_0(s(\hat{\beta}, r^*), r^*) + \underline{u}$$

where the starred variables refer to the contract terms offered in equilibrium. For sophisticated agents, $\hat{\beta} = \beta$ and thus $U_0(P^*) = \underline{u}$ due to the provider's monopolistic power. Using total differentiation of $U_0(P^*)$:

$$\begin{aligned}
\frac{dU_0(P^*)}{d\hat{\beta}} &= \frac{dV_0(P^*)}{d\hat{\beta}} - \frac{d\hat{V}_0(P^*)}{d\hat{\beta}} = \frac{\partial V_0}{\partial r^*} \frac{dr^*}{d\hat{\beta}} - \frac{\partial \hat{V}_0}{\partial r^*} \frac{dr^*}{d\hat{\beta}} - \frac{\partial \hat{V}_0}{\partial \hat{s}} \frac{d\hat{s}}{d\hat{\beta}} = \\
&= \underbrace{-\frac{\partial \hat{V}_0}{\partial \hat{s}} \frac{d\hat{s}}{d\hat{\beta}}}_{\text{the 'distributional effect'}} + \underbrace{\left\{ \frac{\partial V_0}{\partial r^*} - \frac{\partial \hat{V}_0}{\partial r^*} \right\} \frac{dr^*}{d\hat{\beta}}}_{\text{the 'efficiency effect'}}
\end{aligned}$$

Referring to the regularities derived above, both effects have a negative impact on the attained utility.

Finally, it is straightforward to notice that the firm's profits are also increasing in $\hat{\beta}$, as $f^* = [V_0(s(\hat{\beta}, r), r) - \underline{u}]$ is increasing in $\hat{\beta}$ for every r . The firm's optimal choice of (exploitative) r^* only adds to this effect.

Endogenous valuation of the outside option

In the baseline model, the valuation of the agent's outside option is assumed to be exogenous and type-independent, and therefore the 'individual rationality constraint' in the firm's problem of contract design takes the following form:

$$V_0(s(\hat{\beta}, r), r) - f \geq \underline{u},$$

where the first element is also denoted \hat{V}_0 for brevity. Then, taking into account the other constraint $s = s(\hat{\beta}, r)$, the firm's problem can be written concisely as:

$$\max_r V_0(s(\hat{\beta}, r), r) - \underline{u} - c(r, s(\hat{\beta}, r))$$

as in the derivation of Proposition 1.

In contrast to this baseline case, suppose that the agent's outside option involves saving into a costless savings account with a rate of return \underline{r} , e.g. offered by a bank. Then the valuation of the outside option becomes type-dependent:

$$\underline{u} = V_0(s(\hat{\beta}, \underline{r}), \underline{r}) - 0$$

It follows immediately from Lemma 1 that naifs overvalue their outside option, just as any other savings contract. Furthermore, the firm's problem of contract design takes the following form:

$$\max_r V_0(s(\hat{\beta}, r), r) - V_0(s(\hat{\beta}, \underline{r}), \underline{r}) - c(r, s(\beta, r))$$

Note the following. First, as $\frac{d\hat{V}_0}{dr} > 0$, the provider may charge a positive fee only if he offers $r > \underline{r}$.⁴¹ Second, the firm's choice of the optimal rate of return r is independent of the 'individual rationality constraint', which determines the agent's willingness to pay. Consequently, the efficiency properties of the exploitative contracts are the same as predicted by the baseline model. Third, regarding the agent's welfare, the negative efficiency effect prevails under the endogenous valuation of the outside option, but the negative distributional effect is mitigated. In equilibrium, the utility obtained by the agent is:

$$\begin{aligned} U_0(P^*) &= V_0(s(\beta, r^*), r^*) - f^* = V_0(s(\beta, r^*), r^*) - V_0(s(\hat{\beta}, r^*), r^*) + \\ &\quad + V_0(s(\hat{\beta}, \underline{r}), \underline{r}) \end{aligned}$$

While sophisticated agents again obtain the same utility as from their outside option, the impact of naiveté on the attained welfare is derived using total differentiation. This takes the following form:

$$\begin{aligned} \frac{dU_0(P^*)}{d\hat{\beta}} &= \\ &\quad - \underbrace{\frac{\partial V_0(s(\hat{\beta}, r^*), r^*)}{\partial s(\hat{\beta}, r^*)} \frac{ds(\hat{\beta}, r^*)}{d\hat{\beta}}}_{\text{the 'original' distributional effect}} + \underbrace{\left\{ \frac{\partial V_0(s(\beta, r^*), r^*)}{\partial r^*} - \frac{\partial V_0(s(\hat{\beta}, r^*), r^*)}{\partial r^*} \right\} \frac{dr^*}{d\hat{\beta}}}_{\text{the 'original' efficiency effect}} + \end{aligned}$$

⁴¹ $\frac{d\hat{V}_0}{dr} > 0$ holds in the neighbourhood of $\theta = 1$. Naturally, $\frac{\partial \hat{V}_0}{\partial r} > 0$ everywhere, but bear in mind that $\frac{d\hat{V}_0}{dr} = \frac{\partial \hat{V}_0}{\partial r} + \frac{\partial \hat{V}_0}{\partial \hat{s}} \frac{d\hat{s}}{dr}$ and the sign of the second term changes around $\theta = 1$.

$$+ \underbrace{\frac{\partial V_0(s(\hat{\beta}, \underline{r}), \underline{r})}{\partial s(\hat{\beta}, \underline{r})} \frac{d s(\hat{\beta}, \underline{r})}{d \hat{\beta}}}_{\text{the mitigating distributional effect due to overvaluation of } \underline{u}}$$

Moreover, the total distributional effect for naïve agents is positive for $\theta > 1$. Due to the overvaluation of their outside option, naifs do not overpay for their savings contracts, but actually require lower fees than sophisticates in order to satisfy their ‘individual rationality constraint’. The intuition behind this result is as follows. Recall that for $\theta > 1$, naifs overvalue cheaper contracts more, which is captured by $\frac{d^2 \hat{V}_0}{d \hat{\beta} dr} < 0$. In that case, naifs’ overvaluation of the ‘cheap’ outside option $\underline{u} = V_0(s(\hat{\beta}, \underline{r}), \underline{r})$ is of greater magnitude than the overvaluation of the savings contract P^* . As a result, the total distributional effect due to naiveté turns positive.

However, note that under competitive environments there is no role for the distributional effect. Then, naifs are strictly worse off than sophisticates as only the negative efficiency effect prevails.

Policy interventions

Ceiling on fees

Consider the firm’s problem of contract design after an imposition of a ceiling on fees \bar{f} :

$$\max_{r, f} \pi = f - c(r, s), \text{ s.t.:}$$

1. $s = s(\beta, r)$
2. $\hat{V}_0 - f \geq \underline{u}$
3. $f \leq \bar{f}$

The additional constraint affects the optimal contracts by making them (weakly) ‘cheaper’. That is because the modified problem of the firm is:

$$\max_r V_0(s(\hat{\beta}, r), r) - \underline{u} - c(r, s(\beta, r)), \text{ s.t.:}$$

$$1. V_0(s(\hat{\beta}, r), r) - \underline{u} \leq \bar{f}$$

and Lemma 2 follows from the fact that $\frac{d\hat{V}_0}{dr} > 0$ (in the neighbourhood of $\theta = 1$) combined with the derivation of Proposition 1.

Lemma 2 *Under an effective ceiling on fees:*

- *All interest rates are revised downwards.*
- *The efficiency of sophisticated agent’s contracts declines, but consumer welfare is preserved.*
- *Efficiency and consumer welfare attained by naïve agents improve for $\theta < 1$, but decline for $\theta > 1$.*
- *Firm’s profits decrease.*

Therefore a binding ceiling forces the contracts for sophisticated agents away from the first-best. As a result, efficiency decreases, but the consumer welfare is retained as sophisticates obtain their reservation utility \underline{u} . For naïve agents, on the other hand, a decrease in r^* mitigates the adverse efficiency effect, but only for $\theta < 1$. For $\theta > 1$, the exploitative contracts are inefficiently cheap in the first place and an imposition of a ceiling further worsens their efficiency properties as well as makes

naifs worse off compared to the baseline case of no ceiling.

Lastly, note that the presence of the additional constraint lowers firm's profits at the optimum.

Minimum savings requirement

Under a minimum savings requirement \underline{s} , the agent's actual saving is given by $s = \max \{s(\beta, r), \underline{s}\}$, while his expected saving is given by $\hat{s} = \max \{s(\hat{\beta}, r), \underline{s}\}$. Note that this still implies $\hat{s} \geq s$. As a result, there will exist values for r such that $s = \underline{s}$, while $\hat{s} > \underline{s}$. In such situations, the minimum savings requirement is binding, but the agent does not realise it and treats \underline{s} as inconsequential.

The firm's problem of contract design is written as:

$$\max_{r,f} \pi = f - c(r, s), \text{ s.t.:$$

1. $s = \max \{s(\beta, r), \underline{s}\}$
2. $\hat{V}_0 - f \geq \underline{u}$

Whether or not the requirement is binding in equilibrium, and whether the agent realises this, depends on r^* . So how does the imposition of \underline{s} affect the firm's choice of r ? First, note that for $\theta < 1$ and for each r , the provider's marginal cost, which is given by:

$$\frac{dc}{dr} = \frac{\partial c}{\partial r} + \frac{\partial c}{\partial s} \frac{ds}{dr}$$

is bounded from above by the original marginal cost function. That is because $\frac{ds}{dr} > 0$ and the second component of the above disappears in the region for r where the requirement binds. Similarly, the marginal cost is bounded from below by the

original marginal cost function for $\theta > 1$.

Second, the provider's marginal benefit is:

$$\frac{d\hat{V}_0}{dr} = \frac{\partial \hat{V}_0}{\partial r} + \frac{\partial \hat{V}_0}{\partial \hat{s}} \frac{d\hat{s}}{dr}$$

and the above expression is bounded from above (below) by the original marginal benefit function for $\theta < 1$ ($\theta > 1$). Taken together, this implies that in general the introduction of the minimum savings requirement has an ambiguous impact on the firm's choice of the interest rate.

However, as $\hat{\beta} \rightarrow 1$, $\frac{\partial \hat{V}_0}{\partial \hat{s}} \rightarrow 0$ because the agent's period-1 self is believed to be taking the optimal savings decision (from the perspective of period 0). Then, the introduction of the minimum savings requirement does not shift the marginal benefit curve.⁴² Thus in the neighbourhood of $\hat{\beta} = 1$ a more specific prediction can be made. The prevailing shift in the marginal cost curve implies that the offered interest rates are (weakly) higher under the minimum savings requirement for $\theta < 1$ and (weakly) lower for $\theta > 1$. Consequently, the introduction of the minimum savings requirement only makes the extent of exploitation of naïve agents worse. Recall that in the baseline case, the exploitative contracts unambiguously induced over-saving by naifs (relative to sophisticated present-biased agents). Therefore enforcing even higher levels of saving, without affecting the beliefs, cannot lead to improved market outcomes. The above is summarised in Lemma 3.

Lemma 3 *The impact of a minimum savings requirement on optimal interest rates is ambiguous in general. In the neighbourhood of $\hat{\beta} = 1$, however, the minimum savings requirement (weakly) exacerbates the exploitative contract characteristics, which reduces efficiency and consumer welfare.*

⁴²That is true provided that the policy does not force the agents to save 'too much', i.e. more than a time-consistent individual would save.

Competition

Perfect competition

Predictions of the baseline model are derived for a special case of monopoly. Consider another extreme example, namely perfect competition. Under perfect competition, many homogeneous providers freely enter the market, increasing the (endogenous) value of the agent's outside option until each firm's profits are zero at the optimum. The firm's problem of contract design can be thus written as:

$$\max_{r,f} \pi = f - c(r, s), \text{ s.t.:$$

1. $s = s(\beta, r)$
2. $\hat{V}_0 - f \geq \underline{u}$

with the zero-profit condition $\pi^* = 0$ (ZP) holding in equilibrium. Notice that conditional on the first constraint, the second constraint implies

$$f \leq \hat{V}_0 - \underline{u}$$

for any r , while ZP implies:

$$f^* = c(r^*, s(\beta, r^*))$$

in market equilibrium. In a sense, the second constraint determines an upper bound for the fee f , while ZP gives a lower bound, which is binding under the assumption of perfect competition.

Suppose that the firm's profits under monopoly were strictly positive. This implies that under perfect competition the second constraint is slack, while ZP is binding. Then, the equilibrium contracts for each agent-type solve the following problem:

$$\max_r V_0(s(\hat{\beta}, r), r) - c(r, s(\beta, r))$$

As opposed to the case of monopoly, this is not an immediate observation. First, due to ZP all equilibrium contracts make zero profits, so fees are necessarily set equal to $c(r, s(\beta, r))$. Then, the above expression gives the agent's *perceived* utility from a contract offer characterised by a particular interest rate r and a corresponding zero-profit fee. A savings contract that does not maximise the above will not be offered in equilibrium, as a competing firm could offer contract terms that improve the agent's utility while holding the profits constant (at zero).

This has a couple of implications. Importantly, the qualitative nature of inefficient distortions to naifs' contract terms is the same as under monopoly. While sophisticated agents obtain efficient contracts, the direction of the distortions to contracts for naïve agents is again captured by $\frac{d^2 \hat{V}_0}{d\hat{\beta} dr}$. That is because, just as in the baseline model, the equilibrium contract for sophisticates satisfies:

$$\frac{d V_0(s(\beta, r), r)}{dr} = \frac{d c(r, s(\beta, r))}{dr}$$

while the equilibrium contract for naifs satisfies:

$$\frac{d V_0(s(\hat{\beta}, r), r)}{dr} = \frac{d c(r, s(\beta, r))}{dr}$$

Thus the equilibrium interest rates r^* are not affected by the introduction of free entry into the market. However, under perfect competition all agent-types pay strictly lower fees f^* , which makes them better off at the expense of lower profits of the providers.

What is more, since naifs' fees are set so as to cover the *actual* cost of the service $c(r, s(\beta, r))$, naïve agents no longer overpay for their contract terms. In sum, competition does away with the negative distributional effect, but not with the efficiency effect. This can be seen more directly by considering the resulting welfare for each agent-type:

$$U_0(P^*) = V_0(s(\beta, r^*), r^*) - f^* = V_0(s(\beta, r^*), r^*) - c(r, s(\beta, r^*))$$

The above expression is strictly greater than \underline{u} for all agent-types under the assumption of strictly positive monopoly profits. The impact of naiveté on obtained welfare is now captured by:

$$\frac{dU_0(P^*)}{d\hat{\beta}} = \left[\frac{dV_0}{dr^*} - \frac{dc}{dr^*} \right] \frac{dr^*}{d\hat{\beta}} < 0$$

which does include the negative efficiency effect, but not the distributional effect. The fact that this expression is negative follows from the direction of the distortions to r^* and the fact that $\frac{dV_0}{dr}$ is decreasing in r (in the neighbourhood of $\theta = 1$).

Imperfect competition

As noted above, only the fee levels differentiate the equilibrium contracts arising in monopolistic and perfectly competitive environments, while the optimal interest rates are unaffected by the degree of competition. One may thus expect that a model of imperfect competition would generate the same optimal rate of return combined with an intermediate fee. Following Heidhues and Kőszegi (2010), consider a Hotelling-style model, which introduces one additional free parameter.

Assume that there are two identical firms, denoted A and B , located at the endpoints of a unit interval. The firms simultaneously choose both parameters of the offered savings contracts, that is the interest rate r^i and the fee f^i where $i \in \{A, B\}$, so as to maximise their expected profits. There is a unit mass of identical agents distributed uniformly along the interval, or, equivalently, a single agent who is *ex ante* equally likely to find himself located at any point along the interval. Agents sign at most one savings contract. An agent located at $x \in [0, 1]$ evaluates the contract offers according to:

$$\hat{U}_0^A = V_0(s(\hat{\beta}, r^A), r^A) - f^A - \xi x$$

$$\hat{U}_0^B = V_0(s(\hat{\beta}, r^B), r^B) - f^B - \xi(1 - x)$$

where $\xi \geq 0$ is a parameter capturing the agent's 'distance aversion'. In the context of a choice of a financial product, ξ is perhaps more naturally interpreted as capturing the individual's tendency to prefer a 'default' provider.

In case the agent rejects both contract offers, he obtains reservation utility v .

The intuition that both firms will optimally offer the same rate of return r^* as would be offered under monopoly and perfect competition does indeed extend to this setting. Taking the perspective of firm A , the agent's 'individual rationality constraint' is:

$$\hat{U}_0^A = V_0(s(\hat{\beta}, r^A), r^A) - f^A - \xi x \geq \underline{u}$$

where $\underline{u} = \max \{\hat{U}_0^B; v\}$ is endogenous and determined by the competitors contract offer

$P^B = (r^B, f^B)$, the parameters of which firm A takes as given. Let π_x^A denote the firm A's profits from interacting with the agent located at $x \in [0, 1]$, conditional on the above constraint being satisfied:

$$\pi_x^A = f^A - c(r^A, s(\beta, r^A))$$

Because $\frac{\partial \pi_x^A}{\partial f^A} > 0$, the 'individual rationality constraint' binds at the optimum. This implies that the firm's optimal contract offer solves the following:

$$\max_{r^A} V_0(s(\hat{\beta}, r^A), r^A) - \xi x - \underline{u} - c(r^A, s(\beta, r^A))$$

which implies the same r^* as under monopoly and perfect competition, independent of the agent's location x and the parameter ξ . This is also true for the optimal choice of r^B by firm B . As a result, $V_0(s(\hat{\beta}, r^*), r^*)$ is common for both contract offers.

Given the fees charged by both firms, the indifferent agent is located at $x^* \in [0, 1]$, where x^* solves:

$$-f^A - \xi x^* = -f^B - \xi(1 - x^*)$$

or $x^* = \frac{f^B - f^A + \xi}{2\xi}$. This means that individuals located at $x \in [0, x^*]$ obtain greater utility from selecting firm's A offer, and thus the firm attracts a mass x^* of agents when charging f^A . Conditional on f^B , firm A will choose a fee that maximises expected profits:

$$\mathbb{E} \pi^A = \{ f^A - c(r^*, s(\beta, r^*)) \} \times \frac{f^B - f^A + \xi}{2\xi}$$

The above is maximised by $f^{A*} = \frac{f^B + \xi + c(r^*, s(\beta, r^*))}{2}$. Symmetry of the problem implies that $f^{A*} = f^{B*}$ (or, equivalently, $x^* = \frac{1}{2}$). Thus in equilibrium both firms charge:

$$f^* = \begin{cases} \xi + c(r^*, s(\beta, r^*)) & \text{for } \xi \in [0, f^M - c(r^*, s(\beta, r^*))] \\ f^M & \text{for } \xi > f^M - c(r^*, s(\beta, r^*)) \end{cases}$$

where $f^M = f^M(v)$ denotes the fee charged by a monopolistic provider. When the agents are not at all 'distance averse' (i.e. $\xi = 0$), the Hotelling model induces Bertrand competition and thus competitive pricing. The larger ξ , the greater the monopolistic powers enjoyed by the providers. The equilibrium fees are therefore monotonically increasing in ξ . For high enough ξ , the firms charge monopolistic prices.

Variable fees

Variable fee on savings

Suppose that in addition to the flat-rate fee f , the provider is allowed to charge a variable fee t proportional to the agent's savings. The fee is calculated based on the agent's *actual* savings behaviour. The modified problem of contract design is then:

$$\max_{r,f,t} \pi = f + ts - c(r, s), \text{ s.t.:}$$

1. $s = s(\beta, r, t)$
2. $V_0(s(\hat{\beta}, r, t), r, t) - f \geq \underline{u}$

where

$$s(\beta, r, t) = \frac{1}{(1+t)+\beta^{\frac{-1}{\theta}}(1+r)^{\frac{\theta-1}{\theta}}(1+t)^{\frac{1}{\theta}}} \times Y$$

$$V_0(s(\hat{\beta}, r, t), r, t) = \frac{[Y-s(\hat{\beta}, r, t) \times (1+t)]^{1-\theta} - 1}{1-\theta} + \frac{[(1+r) \times s(\hat{\beta}, r, t)]^{1-\theta} - 1}{1-\theta}$$

It is straightforward to observe:

$$\frac{ds(\beta, r, t)}{dt} < 0 \quad \text{and} \quad \frac{d\hat{V}_0}{dt} = \underbrace{\frac{\partial \hat{V}_0}{\partial t}}_{<0} + \underbrace{\frac{\partial \hat{V}_0}{\partial \hat{s}}}_{\geq 0} \underbrace{\frac{d\hat{s}}{dt}}_{<0} < 0$$

The second equation above implies that the presence of variable fees ($t > 0$) reduces the agent's valuation of the contract not only due to the direct effect, but also because variable fees unambiguously reduce his future saving.

After substitution of the binding constraints into the firm's problem, the optimal interest rate satisfies the first order condition:

$$\frac{dV_0(s(\hat{\beta}, r, t), r, t)}{dr} + t \frac{ds(\beta, r, t)}{dr} = \frac{dc(r, s(\beta, r, t))}{dr}$$

An immediate observation is that the agent's naiveté about his present bias, captured by parameter $\hat{\beta}$, affects the optimal interest rate only via $\frac{dV_0(s(\hat{\beta}, r, t), r, t)}{dr}$. That implies that the qualitative properties of the exploitative distortions to contract terms carry

over from the baseline model. In addition, the presence of the variable fee t shifts the optimal interest rate r^* in the direction which increases the agent's actual savings, that is:

$$\frac{dr^*}{dt} \begin{cases} > 0 & \text{for } \theta < 1 \\ = 0 & \text{for } \theta = 1 \\ < 0 & \text{for } \theta > 1 \end{cases}$$

Intuitively, when the provider charges proportionally to the agent's savings, there is an additional incentive to induce higher savings.

As for the optimal choice of the variable fee, t^* satisfies:

$$\frac{dV_0(s(\hat{\beta}, r, t), r, t)}{dt} + s(\beta, r, t) + t \frac{ds(\beta, r, t)}{dt} = \frac{\partial c(r, s(\beta, r, t))}{\partial s(\beta, r, t)} \frac{ds(\beta, r, t)}{dt}$$

This condition implies the existence of inefficient distortions to t^* due to the agent's naiveté. However, in order to determine the sign of $\frac{d^2 \hat{V}_0}{d\hat{\beta} dt}$, and thus the direction of these distortions, one would need to impose additional assumptions on parameters, in particular on $\frac{\partial c(r, s(\beta, r, t))}{\partial s(\beta, r, t)}$ and $\hat{\beta}$. Nonetheless, I conjecture that the direction of such distortions would be independent of the size of the CRRA parameter θ as the expression $\frac{dV_0(s(\hat{\beta}, r, t), r, t)}{dt}$ has a negative sign for all values of t , r , and $\hat{\beta}$.

Variable fee on accumulated wealth

In contrast, suppose that the provider is allowed to charge a variable fee t proportional to the agent's accumulated wealth $(1 + r) \times s$. Then, the firm's problem becomes:

$$\max_{r, f, t} \pi = f + t(1 + r)s - c(r, s), \text{ s.t.:}$$

1. $s = s(\beta, \check{r})$

$$2. V_0(s(\hat{\beta}, \tilde{r}), \tilde{r}) - f \geq \underline{u}$$

where $\tilde{r} = r - t$ approximates the effective net interest rate, $(1 + r)(1 - t) - 1$, and

$$V_0(s(\hat{\beta}, \tilde{r}), \tilde{r}) = \frac{[Y - s(\hat{\beta}, \tilde{r})]^{1-\theta} - 1}{1-\theta} + \frac{[(1+r)(1-t) \times s(\hat{\beta}, \tilde{r})]^{1-\theta} - 1}{1-\theta}$$

For simplicity, the above formulation of the problem ignores the discounting of future revenues by the firm. Following substitution, the optimal interest rate r^* satisfies the following first order condition:

$$\frac{dV_0(s(\hat{\beta}, \tilde{r}), \tilde{r})}{dr} + t\{s(\beta, \tilde{r}) + (1 + r)\frac{ds(\beta, \tilde{r})}{dr}\} = \frac{dc(r, s(\beta, \tilde{r}))}{dr}$$

As an immediate observation, note that the qualitative properties of the distortions to r^* due to $\hat{\beta}$ carry over from the baseline model, as they are captured exclusively by $\frac{d^2 \hat{V}_0}{d\hat{\beta} dr}$. Moreover, in the neighbourhood of $\theta = 1$, the additional term on the left hand side is positive irrespective of $\theta \leq 1$. Consistent with the intuition, the *gross* interest rates are higher when the provider charges proportionally to the size of the agent's pension pot $(1 + r) \times s$.

The optimal variable fee t^* satisfies:

$$(1 + r) \times s(\beta, \tilde{r}) - \frac{dV_0(s(\hat{\beta}, \tilde{r}), \tilde{r})}{d\tilde{r}} - t(1 + r)\frac{ds(\beta, \tilde{r})}{d\tilde{r}} = -\frac{\partial c(r, s(\beta, \tilde{r}))}{\partial s(\beta, \tilde{r})} \frac{ds(\beta, \tilde{r})}{d\tilde{r}}$$

which after plugging in the condition for r^* simplifies to:

$$s(\beta, \tilde{r}) \times (1 + r + t) = \frac{\partial c(r, s(\beta, \tilde{r}))}{\partial r}$$

For the optimally chosen r , the provider's choice of the variable fee t is not directly affected by the degree of the agent's naiveté $\hat{\beta}$.

Financial (il)literacy

‘Headline’ and ‘actual’ fees

Consider the following extension that introduces the possibility of ‘financial illiteracy’ of the agent, albeit in a somewhat crude way. In contrast to naiveté about the present bias (and one’s own behaviour), financially unsophisticated agents misinterpret the contract terms offered by the provider. Suppose that the provider posts a ‘headline’ fee \underline{f} and an ‘actual’ fee \bar{f} . An agent believes that the realised fee will be \underline{f} with probability $\hat{p} \in [0, 1]$, and \bar{f} with residual probability $(1 - \hat{p})$. Without loss of insight, suppose that the realised fee is \bar{f} with probability 1. Thus \hat{p} can be interpreted as a measure of the agent’s financial illiteracy.⁴³ The firm’s problem of contract design is then:

$$\max_{r, \underline{f}, \bar{f}} \pi = \bar{f} - c(r, s) \quad \text{s.t.}:$$

1. $s = s(\beta, r)$
2. $V_0(s(\hat{\beta}, r), r) - \hat{p}\underline{f} - (1 - \hat{p})\bar{f} \geq \underline{u}$

Following substitution, the problem takes the following form:

$$\max_{r, \underline{f}} \quad \frac{1}{(1-\hat{p})} \{V_0(s(\hat{\beta}, r), r) - \hat{p}\underline{f} - \underline{u}\} - c(r, s(\beta, r)) \iff$$

$$\iff \max_{r, \underline{f}} \quad \frac{V_0(s(\hat{\beta}, r), r) - \underline{u}}{(1-\hat{p})} - \frac{\hat{p}}{(1-\hat{p})}\underline{f} - c(r, s(\beta, r))$$

It is clear that the headline fee \underline{f} takes the lowest possible value, for instance $\underline{f} = 0$ due to a likely non-negativity constraint. An offer of a very attractive headline fee

⁴³An example of a more sophisticated model of this kind is Gabaix & Laibson (2006).

allows the provider to charge an accordingly higher actual fee \bar{f} . The optimal interest rate r^* satisfies the following first order condition:

$$\frac{1}{(1-\hat{p})} \times \frac{dV_0(s(\hat{\beta}, r), r)}{dr} = \frac{dc(r, s(\beta, r))}{dr}$$

Under financial illiteracy ($\hat{p} > 0$), the offered interest rates increase. That is because the provider is able to charge a higher *actual* fee for any interest rate. Note that these deviations from what would be the optimal contract terms under $\hat{p} = 0$ should also be interpreted as inefficient. However, the qualitative nature of the exploitative distortions due to agent's naiveté about the present bias, as captured by $\frac{d^2 \hat{V}_0}{d\hat{\beta} dr}$, carries over from the baseline model.

‘Headline’ and ‘actual’ rates of return

Consider an alternative notion of financial illiteracy under which an agent is overly optimistic about the offered rates of return. This could be the case either due to ignorance of any implicit fees or due to optimism about future ‘states of the world’. This is exactly the kind of flaws that seems to underpin the design of increasingly popular structured retail products (Célérier & Vallée 2017). Suppose that the provider posts a ‘headline’ rate of return \bar{r} and an ‘actual’ rate of return \underline{r} . The agent believes that the realised rate will be \bar{r} with probability $\hat{p} \in [0, 1]$, and \underline{r} with residual probability $(1 - \hat{p})$. Without loss of insight, suppose that the realised rate of return is \underline{r} with probability 1. Thus \hat{p} can be interpreted as a measure of the agent's financial illiteracy. The firm's problem of contract design is:

$$\max_{\bar{r}, \underline{r}, f} \pi = f - c(\underline{r}, s) \quad \text{s.t.:}$$

1. $s = s(\beta, \underline{r})$
2. $(1 - \hat{p})V_0(s(\hat{\beta}, \underline{r}), \underline{r}) + \hat{p}V_0(s(\hat{\beta}, \bar{r}), \bar{r}) - f \geq \underline{u}$

The above formulation makes two important implicit assumptions. First, when saving in period 1, the agent is no longer financially unsophisticated, i.e. he takes into account the correct rate of return. As a result, financial illiteracy affects the problem only at the contracting stage. This assumption improves clarity, but is relaxed below. Second, posting two rates of return profits the firm only to the extent that it lowers the cost of the service relative to the collected fees. There is no additional revenue, e.g. due to implicit (but ignored) fees.

Following substitution of the constraints, the problem becomes:

$$\max_{\bar{r}, \underline{r}} (1 - \hat{p})V_0(s(\hat{\beta}, \underline{r}), \underline{r}) + \hat{p}V_0(s(\hat{\beta}, \bar{r}), \bar{r}) - \underline{u} - c(\underline{r}, s(\beta, \underline{r}))$$

Naturally, \bar{r} takes the highest permitted value. This allows the provider to charge higher fees for any actual rate of return \underline{r} . The first order condition regarding the optimal choice of \underline{r} is:

$$(1 - \hat{p}) \times \frac{dV_0(s(\hat{\beta}, \underline{r}), \underline{r})}{d\underline{r}} = \frac{dc(\underline{r}, s(\beta, \underline{r}))}{d\underline{r}}$$

Under this formulation of the model, financial illiteracy (i.e. $\hat{p} > 0$) distorts the equilibrium rates of return downwards, relative to the case of $\hat{p} = 0$. This (inefficient) distortion arises from the fact that the valuation of the contract is now function of the actual, but costly, rate \underline{r} and the costless, but fictitious, rate \bar{r} . However, the qualitative nature of the exploitative distortions due to agent's naiveté about the present bias, captured by $\frac{d^2 \hat{V}_0}{d\hat{\beta} d\underline{r}}$, carries over from the baseline model.

For completeness, consider a slight modification of the model under which the agent is financially unsophisticated (or equally uncertain about the realised interest rate) in both periods 0 and 1. The firm's problem of contract design takes the following form:

$$\max_{\bar{r}, \underline{r}, f} \pi = f - c(\underline{r}, s) \quad \text{s.t.}:$$

1. $s = s(\beta, \bar{r}, \underline{r}, \hat{p})$
2. $(1 - \hat{p})V_0(s(\hat{\beta}, \bar{r}, \underline{r}, \hat{p}), \underline{r}) + \hat{p}V_0(s(\hat{\beta}, \bar{r}, \underline{r}, \hat{p}), \bar{r}) - f \geq \underline{u}$

where $s(\beta, \bar{r}, \underline{r}, \hat{p})$ maximises:

$$\mathbb{E} U_1 = \frac{[Y-s]^{1-\theta}-1}{1-\theta} + \beta \{ \hat{p} \frac{[(1+\bar{r})s]^{1-\theta}-1}{1-\theta} + (1-\hat{p}) \frac{[(1+\underline{r})s]^{1-\theta}-1}{1-\theta} \}$$

Even though there is no closed-form solution for $s(\beta, \bar{r}, \underline{r}, \hat{p})$, the maximisation condition implies that it is continuous in $\hat{p} \in [0, 1]$, ranging from $s(\beta, \underline{r})$ to $s(\beta, \bar{r})$. Analogously for $s(\hat{\beta}, \bar{r}, \underline{r}, \hat{p})$. Following substitution of the constraints, the problem reduces to:

$$\max_{\bar{r}, \underline{r}} (1 - \hat{p})V_0(s(\hat{\beta}, \bar{r}, \underline{r}, \hat{p}), \underline{r}) + \hat{p}V_0(s(\hat{\beta}, \bar{r}, \underline{r}, \hat{p}), \bar{r}) - \underline{u} - c(\underline{r}, s(\beta, \bar{r}, \underline{r}, \hat{p}))$$

Because the headline rate of return now affects the agent's savings, it is no longer true that \bar{r} takes the highest possible value at the optimum. The associated first order condition is:

$$(1 - \hat{p}) \times \frac{\partial V_0(s(\hat{\beta}, \bar{r}, \underline{r}, \hat{p}), \underline{r})}{\partial s(\hat{\beta}, \bar{r}, \underline{r}, \hat{p})} \frac{ds(\hat{\beta}, \bar{r}, \underline{r}, \hat{p})}{d\bar{r}} + \hat{p} \times \frac{\partial V_0(s(\hat{\beta}, \bar{r}, \underline{r}, \hat{p}), \bar{r})}{d\bar{r}} = \frac{\partial c(\underline{r}, s(\beta, \bar{r}, \underline{r}, \hat{p}))}{\partial s(\beta, \bar{r}, \underline{r}, \hat{p})} \frac{ds(\beta, \bar{r}, \underline{r}, \hat{p})}{d\bar{r}}$$

Nevertheless, it is still true that \bar{r}^* is optimally higher than if it was costly to generate for the provider. That is because $\frac{\partial c(\underline{r}, s)}{\partial s} \frac{ds}{d\bar{r}} < \frac{dc(\underline{r}, s)}{d\bar{r}} = \frac{\partial c(\underline{r}, s)}{\partial \bar{r}} + \frac{\partial c(\underline{r}, s)}{\partial s} \frac{ds}{d\bar{r}}$.

The first order condition associated with the actual rate of return \underline{r} is:

$$(1 - \hat{p}) \times \frac{dV_0(s(\hat{\beta}, \bar{r}, \underline{r}, \hat{p}), \underline{r})}{d\underline{r}} + \hat{p} \times \frac{\partial V_0(s(\hat{\beta}, \bar{r}, \underline{r}, \hat{p}), \bar{r})}{\partial s(\hat{\beta}, \bar{r}, \underline{r}, \hat{p})} \frac{ds(\hat{\beta}, \bar{r}, \underline{r}, \hat{p})}{d\underline{r}} = \frac{dc(\underline{r}, s(\hat{\beta}, \bar{r}, \underline{r}, \hat{p}))}{d\underline{r}}$$

As earlier, the possibility of financial illiteracy (i.e. $\hat{p} > 0$) puts a downward pressure on the equilibrium rate of return \underline{r}^* . This observation follows from the fact that

$$\frac{\partial \hat{V}_0}{\partial \hat{s}} \frac{d\hat{s}}{d\underline{r}} < \frac{d\hat{V}_0}{d\underline{r}} = \frac{\partial \hat{V}_0}{\partial \bar{r}} + \frac{\partial \hat{V}_0}{\partial \hat{s}} \frac{d\hat{s}}{d\underline{r}}$$

Importantly, the qualitative impact of naivet  about the present bias on the contract terms carries over from the baseline model (in the neighbourhood of $\theta = 1$).

Imperfect observability with homogeneous beliefs

Consider a mixed population of two indistinguishable agent-types characterised by the same beliefs $\hat{\beta}$. Suppose that share $\lambda \in [0, 1]$ of the agents are present-biased naifs ($\beta < 1$, but $\hat{\beta} = 1$), while share $(1 - \lambda)$ are time-consistent sophisticates ($\beta = \hat{\beta} = 1$). As the two types share their beliefs $\hat{\beta}$, the firm cannot design a separating contract and thus a pooling contract is offered in such a market. The firm's problem of contract design is:

$$\max_{r, f} \mathbb{E} \pi = \mathbb{E} [f - c(r, s)], \text{ s.t.:}$$

1. $s = s(\beta, r)$ with probability λ ; $s = s(1, r)$ with probability $(1 - \lambda)$
2. $V_0(s(1, r), r) - f \geq \underline{u}$

which, following substitution of the constraints, simplifies to:

$$\max_r V_0(s(1, r), r) - \underline{u} - \lambda \times c(r, s(\beta, r)) - (1 - \lambda) \times c(r, s(1, r))$$

The optimal interest rate r^* satisfies the following first order condition:

$$\frac{dV_0(s(1,r),r)}{dr} = \lambda \frac{dc(r,s(\beta,r))}{dr} + (1-\lambda) \frac{dc(r,s(1,r))}{dr}$$

Note that the comparison with the baseline case of perfect observability (isolation), depends on the sign of $\frac{d^2c(r,s)}{ds dr}$. The sign determines whether the provider's marginal cost of offering a higher rate of return is increasing or decreasing in the agent's savings.

- If $\frac{d^2c(r,s)}{ds dr} < 0$, for example due to the scale effects or a greater bargaining power of a pension fund with a larger portfolio, naifs are offered higher interest rates than in isolation. This counteracts the exploitative distortion and improves efficiency and consumer welfare for $\theta > 1$, but only exacerbates the distortion for $\theta < 1$. Sophisticates are offered lower interest rates than in isolation, which unambiguously diminishes efficiency while preserving consumer welfare. That is because sophisticates correctly value their contracts and obtain the utility of \underline{u} .
- If $\frac{d^2c(r,s)}{ds dr} = 0$, then the equilibrium savings contract is as (in)efficient as under perfect observability. Welfare of the two agent-types is not affected by pooling.
- If $\frac{d^2c(r,s)}{ds dr} > 0$, for example due to limits to arbitrage or the liquidity risk, naifs are offered lower interest rates than in isolation, which counteracts the exploitative distortion for $\theta < 1$, but only exacerbates it for $\theta > 1$. Sophisticates are offered higher interest rates than in isolation, which unambiguously diminishes efficiency while preserving consumer welfare.

Imperfect observability with heterogeneous beliefs

In turn, consider a mixed population of two indistinguishable agent-types characterised by different beliefs $\hat{\beta}$. Suppose that share $\lambda \in [0, 1]$ of the agents are present-biased naifs ($\beta < 1$, but $\hat{\beta} = 1$), while share $(1 - \lambda)$ are present-biased sophisticates

($\beta = \hat{\beta} < 1$). The two agent-types are characterised by the same magnitude of the present-bias β . Since the types differ by their beliefs, the firm may design a screening contract. However, depending on the parameter values, it might also be profit-maximising to offer a pooling contract or to exclude the sophisticated agents from the market.

The optimal pooling contract solves the following problem:

$$\max_{r,f} \pi = f - c(r, s), \text{ s.t.:}$$

1. $s = s(\beta, r)$
2. $V_0(s(1, r), r) - f \geq \underline{u}$
3. $V_0(s(\beta, r), r) - f \geq \underline{u}$

Since $\frac{d\hat{V}_0}{d\beta} \geq 0$, constraint 2 (the ‘individual rationality constraint’ for naifs) is slack, while constraint 3 (the ‘individual rationality constraint’ for sophisticates) binds at the optimum. Thus the problem simplifies to:

$$\max_r V_0(s(\beta, r), r) - \underline{u} - c(r, s(\beta, r))$$

Notice that the above describes the optimal contract offered to the sophisticated agents in an isolated market. Such a contract is efficient by Proposition 1, and thus pooling improves both efficiency of naifs’ contract terms as well as their consumer welfare, curbing the negative efficiency and distributional effects. Welfare and efficiency properties of sophisticates’ contracts are the same as under isolation.

A screening contract is a menu $P = \{(r^N, f^N); (r^S, f^S)\}$, which solves:

$$\max_{r^N, r^S, f^N, f^S} \mathbb{E} \pi = \lambda \{f^N - c(r^N, s^N)\} + (1 - \lambda) \{f^S - c(r^S, s^S)\}, \text{ s.t.:}$$

1. $s^N = s(\beta, r^N)$
2. $s^S = s(\beta, r^S)$
3. $V_0(s(1, r^N), r^N) - f^N \geq \underline{u}$
4. $V_0(s(\beta, r^S), r^S) - f^S \geq \underline{u}$
5. $V_0(s(1, r^N), r^N) - f^N \geq V_0(s(1, r^S), r^S) - f^S$
6. $V_0(s(\beta, r^S), r^S) - f^S \geq V_0(s(\beta, r^N), r^N) - f^N$

where superscripts N and S refer to naifs and sophisticates respectively. While constraints 1-4 are standard constraints from the baseline problem, constraints 5 and 6 assure that no agent-type prefers contract terms designed for the other type ('no mimicking').

Fist, notice that the contract terms offered to each agent-type in an isolated market do not solve the above problem. Because $\frac{d\hat{V}_0}{d\beta} \geq 0$, constraint 5 would be violated. As sophisticates do not overpay for their savings contracts, naifs would switch. Conversely, sophisticates would not want to mimic naifs when constraint 3 binds. These observations reduce the problem to:

$$\max_{r^N, r^S, f^N, f^S} \mathbb{E} \pi = \lambda \{f^N - c(r^N, s(\beta, r^N))\} + (1 - \lambda) \{f^S - c(r^S, s(\beta, r^S))\}, \text{ s.t.}:$$

1. $V_0(s(1, r^N), r^N) - f^N \geq V_0(s(1, r^S), r^S) - f^S$
2. $V_0(s(\beta, r^S), r^S) - f^S \geq \underline{u}$

Both of the constraints above bind at the optimum. The firm would like to leave as little rent as possible to naifs, subject to them not mimicking sophisticates. Similarly,

the firm would like to leave as little rent as possible to sophisticates, subject to them accepting the contract offer. Thus constraint 2 implies:

$$f^S = V_0(s(\beta, r^S), r^S) - \underline{u}$$

and constraint 1 implies:

$$\begin{aligned} f^N &= V_0(s(1, r^N), r^N) - V_0(s(1, r^S), r^S) + f^S = \\ &= V_0(s(1, r^N), r^N) - V_0(s(1, r^S), r^S) + V_0(s(\beta, r^S), r^S) - \underline{u} \end{aligned}$$

Substitution reduces the problem to:

$$\begin{aligned} \max_{r^N, r^S} \quad & \lambda \{V_0(s(1, r^N), r^N) - \underline{u} - c(r^N, s(\beta, r^N))\} + \\ & + \lambda \{V_0(s(\beta, r^S), r^S) - V_0(s(1, r^S), r^S)\} + \\ & + (1 - \lambda) \{V_0(s(\beta, r^S), r^S) - \underline{u} - c(r^S, s(\beta, r^S))\} \end{aligned}$$

This formulation implies the following. First, despite screening, naifs are offered the same interest rate as under isolation. Thus imperfect observability does not improve efficiency of their outcomes.⁴⁴ However, due to the no mimicking constraint, naifs pay a lower fee, which improves consumer welfare by curbing the negative distributional effect. Second, the interest rate offered to sophisticated agents is distorted away from the first-best in order to reduce the rent for naïve agents. Thus the efficiency of sophisticates' outcomes deteriorates. However, consumer welfare is preserved.

Lastly, consider the case in which the firm decides to exclude sophisticated agents

⁴⁴This could be interpreted as a version of the well-known 'no distortion at the top' result.

from the market by offering contract terms that violate their ‘individual rationality constraint’, while still satisfying the constraint for naifs. The optimal of such contracts is the one that is offered to naïve agents in an isolated market. Such a contract maximises the firm’s profits from interacting with naifs, but it makes the sophisticated agents select their outside option. In this case, naifs’ outcomes are not affected by imperfect observability, but there is an efficiency loss due to the exclusion of sophisticates from the market. Sophisticated agents, however, retain their welfare.

Pooling, screening and exclusion result in the following equilibrium profits for the provider:

$$\pi^{\text{pooling}} = \pi^S$$

$$\pi^{\text{screening}} = \lambda \tilde{\pi}^N + (1 - \lambda) \tilde{\pi}^S$$

$$\pi^{\text{exclusion}} = \lambda \pi^N$$

where π^S (π^N) denotes the equilibrium profits in an isolated market populated by sophisticates (naifs). $\tilde{\pi}^N$ and $\tilde{\pi}^S$ denote the profits from interacting with naifs and sophisticates respectively under a separating contract. Noting the following:

$$\pi^S \leq \pi^N, \quad \tilde{\pi}^S \leq \pi^S, \quad \tilde{\pi}^N \leq \pi^N$$

shows that either of the three contract design strategies may be profit-maximising, depending on the parameters of the model. In particular, the population composition (λ) and the severity of the present bias (β).

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Chapter 3. Savings Contracts for Naïve Agents: Life-cycle model with price competition

In Chapter 3, I extend the analysis of Chapter 2 by embedding the interaction with a pension provider in a numerical life-cycle framework with hyperbolic discounting. Under the benchmark calibration, the equilibrium contract is Pareto inefficient, lowers the agent's wealth at retirement by 10%, and generates a small, but non-trivial loss of consumer welfare of 0.17% per annum.

3.1 Introduction

The results outlined in Chapter 2 are derived within a simple model that allows for analytical tractability. In Chapter 3, I analyse the impact of exploitative contracting in a more realistic and complex setting by embedding the firm's problem of contract design in a rich life-cycle model with hyperbolic discounting. The theoretical foundation combines the existing work on dynamic choice under present bias with an original model of the supply side of a market for pension products. At the beginning of a life cycle, financial providers simultaneously offer contracts that specify the rate

of return on retirement savings as well as the associated fees. An agent evaluates the contract offers by predicting the resulting consumption and wealth accumulation paths conditional on the contract parameters and his beliefs about the present bias. The Hotelling model of imperfect competition predicts inefficient distortions to the interest rates offered to naïve hyperbolic agents. That is because the contracts that prevail in market equilibrium maximise the *perceived* consumer-firm surplus, rather than the actual surplus. Moreover, the Hotelling model generates a market price that ranges from the perfectly competitive to the monopolistic level according to a single parameter.

To simulate household behaviour, I adopt and modify the numerical life cycle framework of Laibson, Maxted, Repetto and Tobacman (2017).¹ I calibrate the firm side of the model by matching several regularities reported by the empirical literature on operation of pension funds (Bateman, Mitchell 2004; Bikker, de Dreu 2009; Bauer et al. 2010; Bikker et al. 2012; Basu, Andrews 2014; FCA 2017; OECD 2017). The analysis reveals substantial magnitude of the forecasting errors about future saving made by a naïve agent. At the beginning of the life cycle, the agent expects to retire with wealth holdings over twice as large as the actual average wealth holdings. These forecasting errors are reflected in the design of the savings contract offered in market equilibrium. My preferred measure of efficiency is the Pareto criterion which asks whether there exist alternative contract parameters that would improve the agent's welfare, conditional on the magnitude of the present bias and the agent's naiveté, while retaining the profits earned by the firm. Under the benchmark calibration, the equilibrium contract is indeed inefficiently 'cheap', which is consistent with the theoretical prediction. The inefficiency in contract design lowers the agent's wealth

¹As a benchmark case, I use the estimates of the preference parameters that produce the best fit to the US data on wealth accumulation and borrowing, i.e. the short-run discount factor $\beta = 0.505$, the long-run discount factor $\delta = 0.987$, and the CRRA parameter $\theta = 1.255$. These estimates are obtained under the assumption of complete naiveté, i.e. $\hat{\beta} = 1$ (Laibson et al. 2017).

at retirement by 10% and generates a consumption-equivalent welfare loss of 0.17%, or \$173, per annum. This loss of consumer welfare is small, but non-trivial. Its size reflects the conservative efficiency criterion and the fact that the efficient contract increases consumption in retirement, which is heavily discounted in the lifetime utility function. However, note that this result is derived under a constrained contracting space and for a single exploitative contract, constituting a cautious estimate.

Following the calibration of the model, I examine the efficiency and distributional consequences of three common policy interventions - imposing ceilings on fees, regulating the degree of competition, and changing the liquidity of retirement savings. What is noteworthy, interventions that increase the degree of price competition in the market improve efficiency and redistribute wealth from the firm to the agent, while imposing ceilings on charges only diminishes consumer welfare. That is again due to the fact that a binding ceiling precludes the market from providing higher-yielding savings contracts. In addition, while increasing the flexibility of retirement savings exacerbates the overvaluation of savings contracts by a naïve agent, his willingness to pay increases sufficiently for the agent to select the efficient, ‘expensive’ contract offered by the provider.

The remainder of this chapter proceeds as follows. Section 2 presents a dynamic life-cycle model with price competition, the calibration strategy, and the quantitative results, including policy counterfactuals. Section 3 concludes. Derivation of Proposition 2 as well as a full set of the numerical results are relegated to the appendix.

3.2 Life-cycle model with price competition

The baseline model provides a tractable framework within which interactions between a providing firm and a present-biased individual can be studied analytically.

This is a useful first-step exercise as it allows to understand the underlying trade-offs, develop some intuition, and explicitly introduce a range of policy interventions and extensions. However, the consumption-saving choices, and particularly those related to financial preparation for retirement, are more realistically made within a dynamic, complex setting of a life cycle. What is more, developing a numerical life-cycle model with price competition between financial providers allows to evaluate the quantitative implications of the exploitative contract design studied in Chapter 2.

The extended model developed in this section consists of two main building blocks. First, the household side, which is largely based on the existing work on dynamic decision-making under hyperbolic discounting, both theoretical and numerical (Laibson 1994, 1997; Laibson et al. 2000; Harris, Laibson 2001; Diamond, Kőszegi 2003; Laibson et al. 2017; Cao, Werning 2018). Second, the firm side, which constitutes the main contribution of work presented in Chapters 2 and 3. My modelling of the financial providers is guided by the empirical literature on operation of pension funds (Bateman, Mitchell 2004; Bikker, de Dreu 2009; Bauer et al. 2010; Bikker et al. 2012; Basu, Andrews 2014).

This section first sets up a multiperiod model of an interaction between a financial provider and a present-biased agent (household). This formulation of the model constitutes a theoretical foundation for the numerical exercise. Following calibration, I discuss the magnitude of the agent's forecasting errors and the properties of the savings contracts offered in market equilibrium. The forecasting errors deserve attention, as they determine the extent to which the financial providers may exploit the agent's naiveté. Subsequently, the central part of the quantitative analysis regards the impact of contractual design and choice on efficiency, consumer welfare, and savings outcomes. Finally, I consider the impact of three policy interventions

on efficiency of market outcomes and distribution of wealth.

3.2.1 Theoretical foundations

Consider the following model of a market for savings products in a context of a life cycle. Competing financial providers offer savings contracts characterising the key features of the agent's pension asset - the rate of return on accumulated wealth and the underlying fee (price). This is captured by the Hotelling model of competition which, as an equilibrium result, spans all degrees of price competition according to a single parameter.

Regarding the household side of the model, the setup follows the life-cycle framework of Laibson et al. (2017). Consider an economic agent who is alive for a maximum of T periods. Let Y_t for $t \in \{1, 2, \dots, T\}$ denote the agent's disposable income in period t . Each period, the agent chooses his current consumption as well as holdings of illiquid and liquid assets. The illiquid (pension) asset Z is characterised by a constant rate of appreciation R^Z , a (negative) income flow of a fixed value illustrating the fee, and a schedule of time-dependent withdrawal penalties.² The agent is not allowed to borrow against the illiquid asset. The liquid asset X imposes no charges or withdrawal penalties. The agent may hold negative amounts of X up to a certain limit, which illustrates (credit card) borrowing. Consequently, the appreciation rate of the liquid asset depends on whether its holdings are positive or negative. Positive holdings earn a rate of return R^X , while negative holdings generate borrowing costs of R^{CC} . The size of the agent's household changes deterministically throughout the life cycle.

Income and survival are uncertain, and the agent derives utility from leaving a

²The penalty schedule is exogenously given and identical across providers. The modelling of commitment devices constitutes a separate research question (e.g. Amador et al. 2006; Beshears et al. 2014; Bryan et al. 2010).

bequest. In period t , the agent chooses his current consumption and net investments into both assets in order to maximise the expected discounted lifetime utility:

$$U_t = u(C_t) + \mathbb{E}_t \left[\beta \sum_{s=t+1}^T \delta^{s-t} u(C_s) \right],$$

subject to the borrowing and budget constraints (see below). In the above, $u(C_t) = n_t^{\frac{(C_t)^{1-\theta}-1}{1-\theta}}$ with $\theta > 0$ is the instantaneous CRRA utility function which accounts for the household size, denoted n_t . Using the standard notation, $\beta \in (0, 1]$ is a short-run discount factor (the present-bias parameter) and $\delta \in (0, 1]$ is a long-run discount factor. For $\beta = 1$, the decision simplifies to the standard life-cycle optimisation problem with exponential discounting. The expectation operator is taken with respect to the stochastic outcomes, i.e. survival and labour income.

No borrowing on the illiquid asset is captured by the constraint $Z_t \geq 0$, $\forall t \in \{1, 2, \dots, T\}$. The borrowing limit on the liquid asset proportional to the average income at age t is captured by $X_t \geq -\lambda \bar{Y}_t$ for $\lambda > 0$. Let I_t^i denote the agent's net investment into asset i . Then, the dynamic budget constraint associated with the illiquid asset is $Z_{t+1} = (1 + R^Z)(Z_t + I_t^Z)$. The dynamic budget constraint associated with the liquid asset is $X_{t+1} = (1 + R)(X_t + I_t^X)$, where $R = R^X$ if $X_t + I_t^X \geq 0$, and $R = R^{CC}$ otherwise. Let κ_t denote the withdrawal penalty associated with the illiquid asset. Then, the static budget constraint is $C_t = Y_t - I_t^Z - I_t^X + \kappa_t \min(I_t^Z, 0)$.

In a dynamic setting, present bias generates a strategic conflict between subsequent decision-makers, usually referred to as different 'selves' of an economic agent. The conflict arises because of the time-inconsistency inherent to hyperbolic discount functions. The decision-maker in period t discounts utilities in two far-away periods, e.g. $t + s$ and $t + s + 1$, by a long-run discount factor δ . However, when period $t + s$ comes, the decision-maker discounts those utilities by a factor of $\beta\delta$, which is strictly

smaller when $\beta < 1$. Note that such strategic conflict is missing from the simple model presented in Chapter 2, because the model features only one period of saving and because there is no possibility to commit at the contracting stage. Given the time-inconsistency of preferences, the standard equilibrium concept employed by the literature is a pure-strategy subgame-perfect equilibrium. The corresponding equilibrium strategies can be solved for by backward induction and necessarily depend on beliefs regarding behaviour of future selves (Laibson 1994, 1997; Harris, Laibson 2001; Cao, Werning 2018). The following discussion focuses on contrasting two extreme, but illustrative cases - an entirely sophisticated agent ($\hat{\beta} = \beta$) who correctly predicts his future counterparts to be present-biased, and a completely naïve agent ($\hat{\beta} = 1$) who expects his future selves to discount exponentially.

A known issue with the sophisticated dynamic model with hyperbolic discounting is that such a model usually has multiple equilibria. Even though some refinement criteria might ensure uniqueness, the equilibrium consumption functions may be discontinuous due to strategic interactions between various selves (Laibson 1994, 1997). Moreover, standard dynamic programming techniques cannot be used in order to formulate the solution recursively due to the time-inconsistency of the agent's preferences (Laibson et al. 2000). To sidestep these difficulties and be able to develop some intuition, refer to the approximate solution to the dynamic model derived by Harris and Laibson (2001).³ Harris and Laibson show that for $\beta \approx 1$, consumption functions of a sophisticated agent are continuous, in which case the equilibrium behaviour is captured by the following marginality condition:

$$u'(C_t) \geq \mathbb{E}_t \left[(1 + R^Z) \left\{ \frac{\partial C_{t+1}}{\partial Z_{t+1}} \beta \delta + (1 - \frac{\partial C_{t+1}}{\partial Z_{t+1}}) \delta \right\} u'(C_{t+1}) \right]$$

³Cao and Werning (2018) propose an alternative approach by deriving qualitative predictions about the savings behaviour that hold in the whole set of equilibria. What is more, this is achieved without referring to the relevant marginality conditions. However, their results hold only under the case of a stationary infinite-horizon problem. Such approach also cannot deliver similarly robust predictions about consumption.

which is dubbed the (Strong) Hyperbolic Euler Equation. The Hyperbolic Euler Equation differs from the standard Euler Equation by the ‘effective discount factor’ $\frac{\partial C_{t+1}}{\partial Z_{t+1}}\beta\delta + (1 - \frac{\partial C_{t+1}}{\partial Z_{t+1}})\delta$. The effective discount factor is a weighted average of an immediate discount factor $\beta\delta$ and a long-run discount factor δ , where the weights depend on future marginal propensity to consume out of accumulated wealth.⁴ The Hyperbolic Euler Equation holds as an inequality when the borrowing constraint is binding, just as in a standard case of exponential discounting. The above formula is derived for a special case of a single asset with a constant rate of return, no borrowing, and infinite horizon. However, the results of Harris and Laibson (2001) generalise to the case of multiple state variables, including liquid and illiquid assets as well as finite horizon.

On the other hand, strategic interactions do not affect the naïve agent’s decision-making. Since a naif believes that his future selves will act consistently with his intertemporal preference, the choice in period t satisfies the following marginality condition:

$$u'(C_t) \geq \mathbb{E}_t \left[(1 + R^Z) \beta\delta u'(C_{t+1}) \right]$$

while the naif erroneously predicts future selves to act as dictated by:

$$u'(C_{t+s}) \geq \mathbb{E}_{t+s} \left[(1 + R^Z) \delta u'(C_{t+s+1}) \right]$$

⁴Future self over-consumes relative to self- t preferences. The effective discount factor thus has the following intuitive interpretation. The lower future MPC, the greater (i.e. closer to δ) the effective discount factor. That is, self- t endows his future self with more wealth when future behaviour diverges from his preference only to a limited extent.

Note the following regularities which underlie the forecasting errors about future behaviour made by naïve individuals at the contract evaluation stage. First, while sophisticated agents correctly predict their future wealth accumulation path, naifs are overly optimistic about their future saving as they underestimate their future propensity to spend. Second, within any single period, savings of a sophisticated agent constitute an upper bound for naif's savings. In sum, naïve agents are overoptimistic about their utilisation of a savings contract not only because they ignore the impact of the present bias on future choices, but also because their *current* strategies are suboptimal given the prevalence of the bias.⁵

As for the firm side of the model, the financial product underlying the accumulation of the illiquid asset Z is assumed to be provided by a pension fund. As mentioned in the introduction, private pension assets held in the OECD countries have reached the value of \$38 trillion in 2016. Importantly, a major proportion of these assets is managed by pension funds. Pension funds manage 59% of all private pension assets in the US, and this fraction is even higher in other countries with substantial private pension wealth (OECD 2017).⁶ Thus focusing on pension funds allows to study the incentives faced by important and universal managers of pension wealth, who are able to offer tailor-made financial products via retail markets as well as workplace schemes. Note that the characteristics of contractual arrangements between these providers and savers are a crucial determinant of future incomes in retirement. Moreover, as markets for pension products tend to be heavily regulated and the institutional framework varies substantially across countries, the policy im-

⁵Furthermore, with multiple asset classes, the income and substitution effects of the interest rate on optimal savings are not as straightforward. The changes in the rates of return affect also the proportions of total wealth held in illiquid and liquid assets. Moreover, in a dynamic setting, the interest rate affects the timing of the savings decisions. For example, it is possible that under $\theta < 1$ and a higher rate of return, the agent starts saving for retirement earlier in his life cycle. Then, the total (cumulative) amount saved is not necessarily increasing in the interest rate.

⁶Pension funds manage 54% of all private pension assets in Canada, 100% in the UK, 97% in Australia, 61% in Japan, and 100% in the Netherlands.

plications of the following numerical analysis should be of special interest.

Given the increasing role of private pensions in determining incomes in retirement, the empirical evidence on operational costs of pension funds is somewhat limited (Bateman, Mitchell 2004; Bikker, de Dreu 2009; Bauer et al. 2010; Bikker et al. 2012; Basu, Andrews 2014). The existing literature typically divides the total costs of pension funds into administrative and investment costs.⁷ Findings regarding the administrative cost component appear to be consistent across studies. There are significant scale effects. After controlling for the number of participants in a pension plan, the associated administrative costs increase in the size of assets under management, albeit less than proportionately. The estimated elasticity of administrative costs with respect to assets varies from 0.19 to 0.87.⁸ The evidence regarding investment costs is not as clear. Naturally, that is largely due to the inability to equalise *ex post* realised returns of a pension fund with *ex ante* costly effort. Nonetheless, expenses of pension funds have been shown to increase in factors that are typically associated with a higher expected rate of return, i.e. the number of available investment options, the ‘complexity’ of a pension plan, the proportion of assets invested in stocks, and the proportion of actively managed assets (Basu, Andrews 2014; Bauer et al. 2010; Bikker et al. 2012). The finance literature offers arguments in favour and against scale effects in investment costs (with respect to assets under management). The former include fixed costs and greater bargaining power of larger pension funds, while the latter invoke limits to arbitrage and the liquidity risk. To account for these regularities, assume the following per-period cost function of the financial provider:

⁷For example, Bikker and de Dreu (2009) define investment costs as directly related to asset management (e.g. wages of portfolio managers and analysts, trading costs). Administrative costs include all other operational expenses related to record-keeping, communication with participants, marketing, and compliance with existing legal requirements (e.g. wages, rent).

⁸The large discrepancy arises not only as a result of cross-country differences in the setup of markets for pension products, but it also reflects the differences in data quality and definitions of administrative costs across studies.

$$c(R^Z, Z_t) = c_1 Z_t^{\gamma_1} + c_2 (R^Z - R^X)^{\gamma_2}$$

where $c_1, \gamma_1, c_2, \gamma_2 > 0$. In the above cost function, the first term corresponds to administrative costs borne by pension funds, which are increasing in the accumulated assets. The second term corresponds to investment costs, which are increasing in the difference between the offered rate of return R^Z and a ‘risk-free’ rate of return R^X on the liquid asset, which the agent (and supposedly the financial provider) may access costlessly.⁹

Collected fee f constitutes the firm’s revenue, and thus the per-period profit is:

$$\pi_t = f - c(R^Z, Z_t)$$

What would be an appropriate way to model market competition in this context? The empirical literature (Bateman, Mitchell 2004; Dobronogov, Murthi 2005; Tapia, Yermo 2008; Bikker, de Dreu 2009; Bauer et al. 2010; Bikker et al. 2012; Basu, Andrews 2014) makes no attempt to explicitly evaluate the degree of competition in the analysed markets for pension products. That is perhaps due to the fact that a range of factors besides the number of competing firms seem to determine the observed pricing. On the one hand, the available industry reports suggest a low degree of market concentration. For example, the 10 largest pension funds held 8.5% of all private pension assets in the US in 2016, and the 20 largest pension funds in the

⁹The quantitative model presented below simulates an interaction between the household and a ‘representative’ financial provider offering a stylised savings contract. From an empirical perspective, however, this approach is hindered by a couple of limitations. First, it ignores the fact that pension providers operating in seemingly competitive markets differ substantially from one another in terms of cost levels, fees charged, resulting profits margins, and details of the services provided. Indeed, a large within-country heterogeneity among pension providers, and the details of private pension arrangements they offer, is one of the prevalent findings documented by this literature. Second, the characteristics of offered pensions plans tend to vary substantially across countries, reflecting the differences in institutional framework and market maturity. These issues should be carefully addressed in future work.

world managed 17% of all assets (Willis Towers Watson 2017). On the other hand, markets for pension products are not necessarily characterised by competitive pricing, despite no major barriers to entry and an increasing number of providers. This is mainly because of a weak competitive pressure on prices coming from the demand side. The complexity of available products, complicated pricing structures, and inertia in pension choices all contribute to this effect (Office of Fair Trading 2014).¹⁰ In addition, the charged fees reflect the details of a particular regulatory framework, which may include price controls and discourage switching between providers. Given these caveats, I analyse a Hotelling model of market competition which, as an equilibrium result, spans all degrees of price competition according to a single parameter.

Consider a Hotelling model of spatial competition. Two firms, called A and B , are located at endpoints of a unit interval. The firms are otherwise identical. There is a measure 1 of consumers distributed uniformly along the interval. (Equivalently: there is a single consumer who is *ex ante* equally likely to find himself at any point along the interval.) The firms make their contract offers simultaneously and the agents evaluate them at time $t = 0$, that is before the beginning of the life cycle. The savings contracts specify the parameters of the illiquid asset Z - the rate of return on the agent's pension wealth and the fee charged for the service. The contracts are not renegotiable and the parameters bind throughout the entire life cycle. Finally, each consumer signs at most one savings contract.¹¹ An agent located at $x \in [0, 1]$ derives the following *perceived* utilities from signing a contract P with firm A and firm B , respectively:

¹⁰In fact, the cross-country analysis of Tapia and Yermo (2008) links greater competition, as measured by the number of providers, to higher prices. This is driven by the additional marketing expenses which are reflected in the fees.

¹¹Admittedly, optimal timing of the contract offer, the possibility of renegotiation, and extensions to richer contracting spaces all constitute interesting directions for future research.

$$\hat{U}_0(P_A) = V_0(\mathbf{C}(\hat{\beta}, R_A^Z, f_A)) - \xi x$$

$$\hat{U}_0(P_B) = V_0(\mathbf{C}(\hat{\beta}, R_B^Z, f_B)) - \xi(1 - x),$$

where $\hat{U}_0(P)$ is computed by plugging in the expected life-cycle consumption path $\mathbf{C} = (C_1(\hat{\beta}, R^Z) - f, C_2(\hat{\beta}, R^Z) - f, \dots, C_T(\hat{\beta}, R^Z) - f)$ into the lifetime utility function under exponential discounting:

$$V_0 = \mathbb{E}_0 \sum_{s=1}^T \delta^s u(C_s - f)$$

As in Chapter 2, and most of the literature, the ‘unbiased’ utility function constitutes the normative welfare benchmark. The expected consumption path \mathbf{C} depends on the agent’s savings behaviour and as such is a function of the rate of return R^Z associated with the pension asset as well as the present bias parameter $\hat{\beta}$. However, for computational feasibility, I maintain the assumption of a negligible impact of fees on the savings behaviour, i.e. $\frac{dC_t}{df} = 0$.¹² Parameter ξ measures the agent’s ‘distance aversion’. In the context of a market for financial products, this is perhaps more naturally interpreted as the agent’s tendency to select a ‘default’ provider. If an agent rejects contract offers of both firms, he obtains reservation utility v .

The firms design their contracts in order to maximise expected profits. Firm’s i expected profits from an accepted contract are given by:

¹²This avoids re-simulating the behaviour over an entire life cycle for each fee level. In one of the robustness checks, I show that this simplifying assumption has a minimal impact on the optimal consumption paths, both expected and actual.

$$\mathbb{E}_0 \pi_i = \mathbb{E}_0 \sum_{t=1}^T \frac{1}{(1 + R^X)^t} \pi_{i,t}$$

where the expectation operator is taken with respect to the agent's survival and income, as the (unconditional) contract terms are binding throughout his life cycle. It is assumed that the firms discount future profits using the risk-free rate R^X .

The optimisation problem of firm i is:

$$\max_{R_i^Z, f_i} \mathcal{P} \times \mathbb{E}_0 \pi_i \quad \text{s.t.}:$$

1. $Z_t = Z_t(\beta, R_i^Z)$
2. $\mathcal{P} = \mathbb{P}(\hat{U}_0(P_i) \geq \underline{u})$
3. $\mathbb{E}_0 \pi_i \geq 0$

The first constraint, as before, says that when computing the agent's actual wealth accumulation path, which determines the cost of the service, the firm takes into account the true present bias parameter β and the offered rate of return R_i^Z . The second constraint is a version of the 'individual rationality constraint', which regards the probability of the agent accepting firm's i offer given his realised location x . In the above, $\underline{u} = \max \{\hat{U}_0(P_{-i}), v\}$ is dependent on the contract parameters offered by a competitor, which firm i takes as given. The third constraint assures that the firm only enters the market when it earns non-negative profits on average.

The following summarises the predictions of such multiperiod Hotelling model.

Proposition 2 *In a multiperiod Hotelling model of market competition:*

1. *Both firms offer the same interest rate R^{Z*} , which maximises the perceived consumer-firm surplus and is independent of ξ .*
2. *Both firms charge the same fee f^* . The equilibrium fee is monotonically increasing in ξ , ranging from perfectly competitive pricing to monopolistic pricing.*

The derivation is relegated to the appendix. As for an intuitive interpretation of this result, one might think of the parameter ξ as capturing the extent of each firm's monopolistic power. For low values of ξ , the agent's 'distance aversion' is of secondary importance when comparing the contract offers, and he is therefore selecting the more attractive contract terms independently of provider's location. From the firms' perspective, that implies that lowering (raising) the price has a large positive (negative) impact on each firm's market share. For low values of ξ , this induces Bertrand competition and competitive pricing. For greater values of ξ , on the other hand, the agent is more likely to select an offer of his default provider so as to avoid 'travelling'. Once the agent becomes extremely distance-averse, the firms can charge as if they were monopolists. Thus, as an equilibrium result, parameter ξ determines the degree of price competition in the market.

Similarly to the baseline model, the degree of competition does not affect the choice of the interest rate R^{Z*} . Both firms select the interest rate that maximises the *perceived* consumer-firm surplus, rather than the actual surplus.

Note that the above Hotelling formulation is chosen primarily for the sake of tractability. There is a single parameter corresponding to the degree of price competition, and a single equilibrium price. Thus the Hotelling model is my preferred foundation for the numerical application, in which the household interacts with a

‘representative’ financial provider. However, given the substantial and persistent price dispersion in markets for pension products, future work should consider models that are capable of generating a distribution of market prices.¹³ Notice also that the above formulation of the model no longer assumes quasi-linearity of the utility function. This has a non-trivial impact on the notion of ‘efficiency’. The simplifying assumption of quasi-linearity, made by a majority of standard models, implies the existence of a common measure of ‘social surplus’, which both the firm and the agent would wish to maximise. However, under a setup in which the agent’s preferences are concave in the fees paid (as all kinds of wealth are implicitly assumed to be fully fungible), any chosen measure of ‘social surplus’ would be affected by the size of the transfer from the agent to the risk neutral firm. In other words, defining an efficiency criterion without the assumption quasi-linearity usually requires the modeller to specify the relative weights that the criterion attaches to the agent’s utility and the firm’s profits. Lastly, compared to the textbook formulations of a model of spatial competition (Tirole 1989), in the above the two firms are not able to select their locations, but choose a ‘product quality’ instead.

3.2.2 Calibration

To model the agent’s expected and actual behaviour, I modify and extend a rich life-cycle framework with hyperbolic discounting developed by Laibson, Maxted, Repetto and Tobacman (2017), henceforth LMRT.¹⁴ The setup of the numerical framework

¹³In the present context, models of costly consumer search may be particularly useful. The classical contributions to this literature (Butters 1977; Burdett, Judd 1983; Stahl 1989) analyse models in which price dispersion arises in a market for a homogeneous good. Hortacsu and Syverson (2004) use one variant of such models in order to explain price dispersion in a market for index funds in the US, which are, by definition, homogeneous. There are also behavioural models that predict price dispersion in absence of products differentiation or complicated pricing. Examples includes models of sampling-based reasoning and models with status quo bias (Spiegler 2014).

¹⁴The numerical model of Laibson et al. (2017) is the most recent iteration of earlier models of Laibson et al. (2000) and Angeletos et al. (2001).

follows the model of a present-biased household outlined in section 3.2.1. Survival and income processes are stochastic and calibrated to the relevant data for the US. Time-varying household size and retirement age are deterministic, and also calibrated to the US data. I adopt the specifications for survival and income processes, household size, and retirement age that are calibrated by LMRT.¹⁵

Households hold two kinds of wealth - liquid and illiquid assets. I follow the benchmark parametrisation of the liquid asset X in LMRT. The positive holdings of X generate returns of $R^X = 2.79\%$, while the negative holdings generate borrowing costs of $R^{CC} = 11.52\%$.¹⁶ However, I modify the way in which the illiquid asset is modelled so that it resembles pension wealth more closely. Under the benchmark assumptions of LMRT, the illiquid asset generates no capital earnings, but provides a proportional consumption flow of 5%. The associated liquidation costs decrease monotonically with age. LMRT acknowledge that under these assumptions, the illiquid asset shares some features with housing as well as with pension wealth. In contrast, I model the illiquid asset Z that appreciates over time at a rate R^Z , but generates no additional consumption flow. I also modify the specification of the liquidation costs so that they are high during the agent's working life, but decrease sharply at retirement.¹⁷

LMRT develop the structural model in order to estimate the discount factors and the CRRA parameter by matching the data on wealth accumulation and credit card

¹⁵In particular, the disposable income process is defined as a sum of a polynomial in age, an AR(1), and a random shock. The parameters of the income process are estimated using the PSID data. Retirement occurs exogenously at the age of 64.

¹⁶ R^X is equal to a long-run average of real yields on AAA bonds. The choice of R^{CC} reflects the average quarterly interest rate reported by the Fed, bankruptcy rate, and inflation.

¹⁷The liquidation costs evolve over time according to the function $0.5 \times \frac{1}{1+e^{\frac{\text{age}-61}{2}}}$, which starts at the value of 0.5 at young age and then decreases monotonically. Compared to the original function assumed by LMRT, $0.5 \times \frac{1}{1+e^{\frac{\text{age}-50}{10}}}$, these liquidation costs start decreasing closer to the retirement age, but decrease more sharply. The crossing point of the two functions coincides with the calibrated timing of the agent's retirement (age of 64).

borrowing among US households using the Method of Simulated Moments (Gourinchas, Parker 2002).¹⁸ For their benchmark specification, the best match to the data is achieved by setting $\beta = 0.505$, $\delta = 0.987$, and $\theta = 1.255$. Due to greater numerical stability, the calculations are performed under the assumption of (complete) naiveté of a hyperbolic household, i.e. $\hat{\beta} = 1$. However, LMRT point out that the quantitative results remain similar for the case of sophistication, see also the discussion in Angeletos et al. (2001). I set the household preference parameters equal to the values estimated by LMRT, which allows to generate realistic wealth accumulation paths that are consistent with the US data. The adoption of these parameters is justified by the fact that I modify the economic environment only to a limited extent relative to the original framework, by changing certain features of the illiquid asset but maintaining all other assumptions. To model the (erroneously) expected wealth accumulation paths, I maintain the assumption of complete naiveté and simulate an otherwise identically parametrised household with $\beta = 1$.

Introduction and calibration of the firm side of the model are the main contributions of the following numerical exercise. I take parameter γ_1 , the elasticity of administrative costs, directly from the empirical literature. The remaining four free parameters of the firm side of the model (c_1, c_2, γ_2, ξ) are calibrated jointly to meet the numerical targets regarding cost-to-assets ratio, share of administrative costs in the total cost, and firm's markup, and to target the interest rate offered in the equilibrium. As a benchmark case, I target the cost-to-assets ratio of 0.005, the share of administrative costs of 0.50, the markup of 0.20, and the equilibrium interest rate of 5%. Parameter γ_1 , the elasticity of administrative costs, is set equal to 0.5 (Bateman, Mitchell 2004; Bauer et al. 2010; Bikker, de Dreu 2009; Bikker et al. 2012; FCA

¹⁸The data used by LMRT comes from the Surveys of Consumer Finances (1989-2013). The targeted moments are the fraction of households borrowing on their credit card, borrowing relative to income, and wealth relative to income, by 10-year age categories.

2017; OECD 2017). The choice of these target values is discussed in more detail in the appendix. Where applicable, all numerical values are calculated at the means.

The market equilibrium is found numerically in the following way. I allow the firm to offer a range of rates of return from a grid centred around the target value R^{Z*} . Specifically, I consider a grid of nine, equally spaced values $R^Z \in \{4\%, 4.25\%, \dots, 5.75\%, 6\%\}$, as the benchmark target is $R^{Z*} = 5\%$. The grid is somewhat narrow, but, as noted by LMRT, the estimates of the household preference parameters are sensitive to the calibration of the economic environment. Analysing household behaviour for a wider grid of values may well generate unrealistic wealth accumulation paths. For each constellation of parameters which satisfies the costs and the markup targets at R^{Z*} , I check whether there exist any contract offers with alternative interest rates (and fees) that increase the agent's *perceived* utility while holding firm's profits at least constant. If there are no such 'mutually preferable deviations', R^{Z*} is indeed offered in market equilibrium. Otherwise, a different level of the interest rate would prevail in the market.

Intuitively, while parameters c_2 and γ_2 jointly determine the cost-to-assets ratio, parameter c_1 is calibrated to match the share of administrative costs and parameter ξ allows to match the requested markup (the greater ξ , the greater the firm's monopolistic power and the equilibrium fees). To rule out preferable deviations, the cost function needs to be steep (convex) enough. Thus the condition for the market interest rate R^{Z*} determines γ_2 .

Table 3.2.1 reports the values of jointly calibrated parameters, corresponding target moments, and the set parameters. For illustration, note that the resulting equilibrium fee is equal to 36% of a monopolistic fee that would extract the entire perceived consumer surplus.

Table 3.2.1: Calibrated parameters

Jointly calibrated parameters	Value	Target moment	Moment value
Admin cost multiplier c_1	1.3797	Share of admin costs	0.50
Investment cost elasticity γ_2	5.75	Market interest rate	5%
Investment cost multiplier c_2	2.19×10^{12}	Cost-to-assets ratio	0.005
Hotelling parameter ξ	0.0808	Markup	0.20
<hr/>			
Set parameters	Value	Source	
Admin cost elasticity γ_1	0.5	Bateman, Mitchell 2004; Bikker et al. 2012	
CRRA parameter θ	1.255	Laibson et al. 2017	
Short-run discount factor β	0.505	Laibson et al. 2017	
Long-run discount factor δ	0.987	Laibson et al. 2017	
Beliefs about present bias $\hat{\beta}$	1	Laibson et al. 2017	
Risk-free interest rate R^X	2.79%	Laibson et al. 2017	

Non-targeted moments

The numerical model replicates well some non-targeted regularities reported by the empirical literature. The equilibrium fee is equal to 0.6% of assets, measured at the mean. This is well within the range of fee levels observed in the pension industry, which are typically between 0.5% and 1.7% (Dobronogov, Murthi 2005; OECD 2017; Tapia, Yermo 2008). The absolute (dollar) value of the calibrated administrative costs, which is equal to \$1250 on average, is somewhat high, but it lies close to the upper end of a range of values reported in the literature.¹⁹ The cost-to-assets ratio, which is calibrated to a value of 0.005 at the means, varies from 0.0035 to 0.0326 over the life cycle, with an average of 0.01, which also falls within the typical range observed across studies (Bateman, Mitchell 2004; Bauer et al. 2010; Bikker, de Dreu

¹⁹Bikker and de Dreu (2009) note a substantial heterogeneity in administrative costs of the Dutch pension funds, which vary from \$53 to \$1509. These bounds are slightly narrower for a sample of larger pension funds from four countries in Bikker et al. (2012), ranging from \$30 to \$674. For the US, Bateman and Mitchell (2004) report the range from \$105 to \$897.

2009; Bikker et al. 2012; OECD 2017).

3.2.3 Quantitative results

Magnitude of the forecasting errors

The forecasting errors regarding future saving enable the exploitation of naïve agents by the financial providers. Furthermore, the magnitude of these errors and the impact of the interest rate thereon, determine the extent of exploitation and the characteristics of the savings contracts offered in market equilibrium. Figure 3.2.1 plots actual and expected wealth holdings over the life cycle, averaged over the income shocks and conditional on survival. The illustrated forecasting errors are interpreted as made by a hyperbolic household at the contract evaluation stage at the beginning of a life cycle.

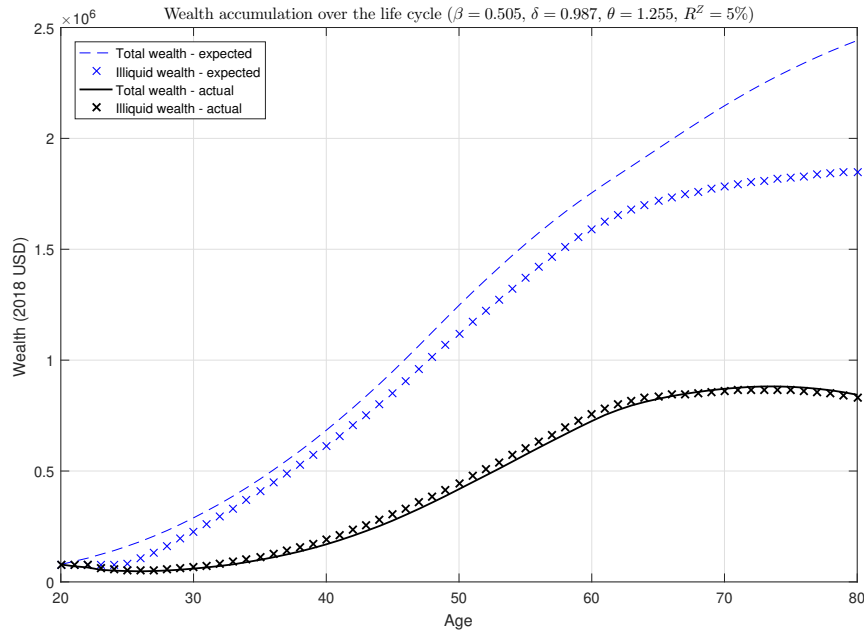


Figure 3.2.1: Benchmark wealth accumulation paths (expected and actual)
Source: Author's calculations

Several immediate observations can be made. Firstly, the magnitude of the forecasting errors regarding future wealth holdings is huge. At retirement at the age of 64, the expected holdings of total as well as illiquid wealth are over two times as high as the actual average wealth holdings. Secondly, and not surprisingly, the period-by-period forecasting errors of a naïve household compound. As a result, the total error increases over time. Thirdly, the composition of total wealth also differs substantially. While a naïve household accumulates virtually all its wealth in a form of the illiquid asset, it erroneously expects to hold some liquid wealth as well. What is more, there are periods early in the life cycle when the value of the illiquid asset exceeds the value of naïf's total wealth. That is because a hyperbolic household simultaneously accumulates illiquid wealth and borrows on its credit card.

Recall that the qualitative implications of the baseline theoretical model rely on the impact of the interest rate on the forecasting errors made by naïve agents. A profit-maximising provider chooses the contract terms that magnify the difference between the agent's forecasted and actual saving behaviour, as such a design leads to a greater overvaluation of a contract by naïfs. In the simple setting, this is achieved by offering inefficiently high interest rates when the substitution effect dominates the agent's utility function, and inefficiently low interest rates otherwise. This underlying mechanism appears to extend to a multiperiod model. Under the benchmark value of $\theta = 1.255$, the forecasting errors regarding future saving are indeed *decreasing* in the interest rate associated with the illiquid asset for $R^Z \in [4\%, 6\%]$.²⁰ As an example, Figure 3.2.2 plots the expected and actual retirement saving over the life

²⁰There are several definitions of a 'forecasting error' that one could assume in this context. In order to correct for differences in the timing of savings decisions (e.g. under higher interest rates, the agents tend to start saving for their retirement earlier on in the life cycle) and to provide a measure that corresponds closely to the variable s from the baseline theoretical model, I define an average forecasting error as a difference between the expected cumulative retirement savings and the actual cumulative retirement savings. This is equivalent to analysing a difference between the average per-period retirement savings.

cycle. The forecasting errors of a naïve household are, on average, larger under the lower interest rate of 4.25%, because the expected saving increases more strongly than the actual saving.

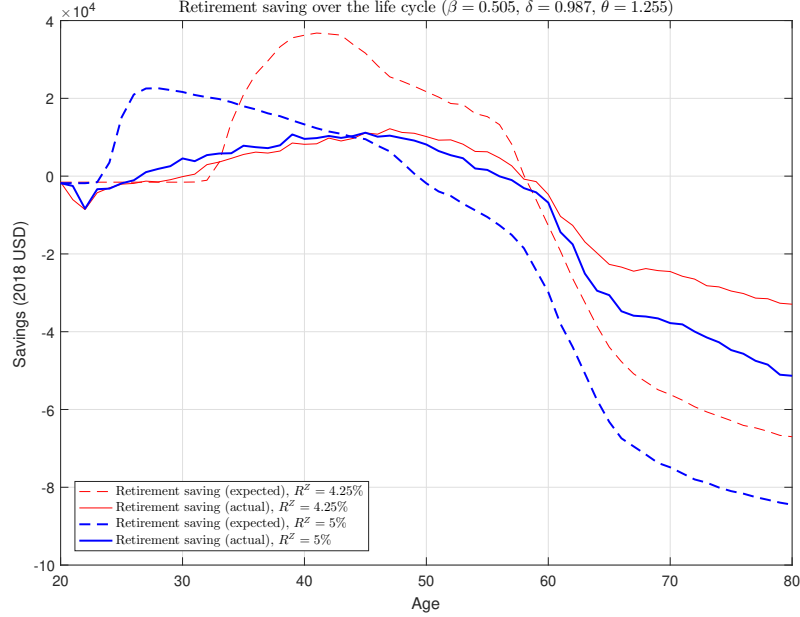


Figure 3.2.2: Retirement saving over the life cycle (expected and actual)
Source: Author's calculations

For an alternative household parametrisation with $\theta = 2.0$, the same pattern holds, i.e. the forecasting errors are decreasing in R^Z . For $\theta = 0.5$, on the other hand, the pattern reverses and the forecasting errors are increasing in R^Z as long as the cumulative savings are increasing as well.²¹

²¹These two alternative values for the CRRA parameter are used by LMRT to check the robustness of their main results. In order to generate realistic wealth accumulation paths, the remaining preference parameters of the household are recalibrated. The estimated δ increases monotonically in θ , and the estimated β decreases monotonically. Moreover, an important caveat applies to the case of $\theta = 0.5$. In a dynamic setting, the interest rate influences not only the amounts saved every period, but also the timing of savings decisions. For values of R^Z higher than 4.5%, the agent starts saving for retirement at earlier ages and as a result does not necessarily save more over the course of the life cycle when R^Z increases. The forecasting errors of a naïve household are increasing in the interest rate as long as the cumulative savings are increasing as well.

Properties of the equilibrium contract

According to the target, the savings contract offered in market equilibrium is characterised by a rate of return $R^Z = 5\%$. Having calibrated the firm's cost function, one can examine whether there exist any alternative Pareto-improving contracts. This is performed in the following way. For alternative interest rates, I calculate fees that would preserve the provider's profits at the current (equilibrium) level. If any of such alternative, profit-preserving contracts improves the agent's *actual* welfare relative to the equilibrium contract, I call it a Pareto improvement. The only reason why there may ever exist Pareto-improving contracts which are not provided by the market are the agent's forecasting errors regarding his future savings behaviour and the resulting wealth accumulation path. Under sophistication, the construction of the market equilibrium would rule out the existence of Pareto improvements.

As the firm's profits are preserved, the Pareto criterion constitutes a conservative efficiency benchmark. More precisely, any other efficiency benchmark that aggregates the agent's utility and the firm's profits with positive relative weights would indicate the existence of inefficiency as long as there exists a Pareto improvement, but not the way around. The advantage of the chosen efficiency benchmark is that the Pareto criterion avoids attaching arbitrary weights to the agent's and the firm's outcomes.

For the benchmark calibration of the model, the Pareto-efficient contract would offer a higher rate of return than the market contract, i.e. $R^Z = 5.25\%$. The profit-maximising contract is in fact inefficiently cheap, consistently with the qualitative prediction of the simple baseline model. Recall that the calibration of the household has $\theta > 1$ and thus the income effect dominates.²²

²²Why is the difference between the efficient and the equilibrium interest rates so small despite substantial forecasting errors made by a naïve household? First, I employ a conservative efficiency criterion. Second, the calibration has θ close to 1, which limits the quantitative impact of the

As a result of selecting a cheap savings contract, the size of the agent's savings pot at retirement is 10% smaller than under the efficient contract. That reflects not only the compounding of the corresponding interest rates, but also the different consumption and wealth accumulation paths induced by the two contracts.²³ As shown in Figure 3.2.3, while the Pareto-efficient contract lowers consumption at the stage of accumulation of pension wealth, it allows higher consumption in retirement. However, it needs to be stressed that the prevalence of the inefficient contract terms in the market is *not* due to preference for immediate consumption, but due to a wrong forecast of future behaviour used as an input into a bias-free contract evaluation.

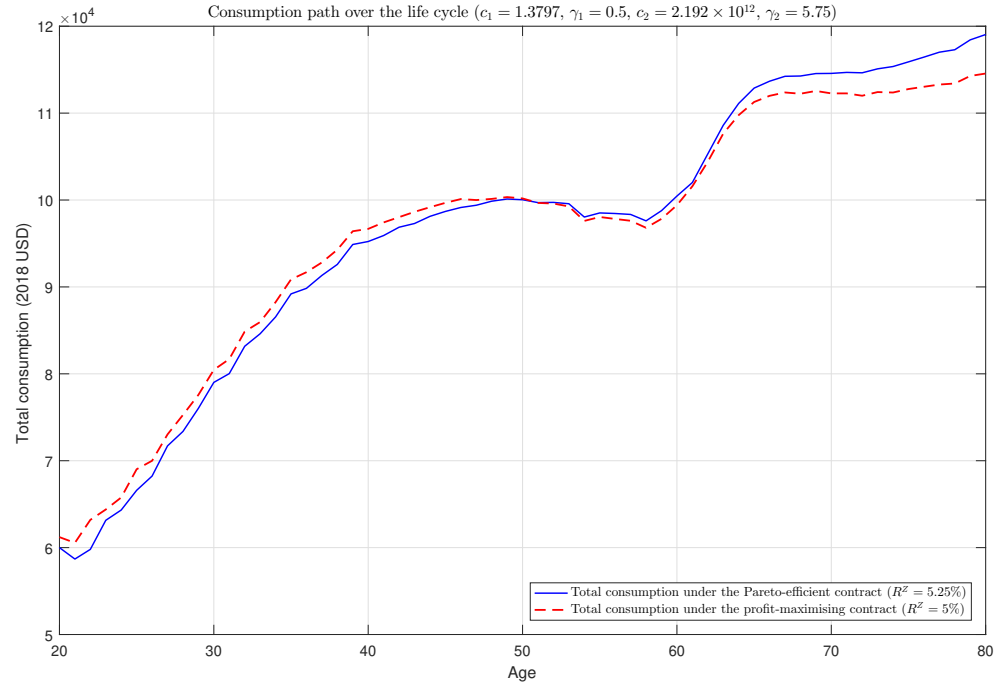


Figure 3.2.3: Savings contracts and consumption paths
Source: Author's calculations

A consumption-equivalent loss of consumer welfare arising from an inefficient interest rate on the extent of overvaluation of a contract.

²³A simple calculation shows that every dollar saved at the age of 40 earning an interest of 5.25% p.a. yields 5.9% higher total returns at the age of 64 when compared to an interest of 5% p.a.

contractual choice is equal to 0.17% per annum. In other words, to be compensated for choosing an inefficient savings contract, the agent's consumption would need to increase by 0.17% in every period of the model. This corresponds to \$173 of foregone consumption annually, which does not increase the provider's profits. Arguably, this estimated loss of consumer welfare arising from exploitative contracting is small, but non-trivial. The modest size of the loss reflects the conservative efficiency criterion, the fact that the CRRA parameter θ is close to 1, and the fact that the efficient contract provides higher consumption in retirement, which is heavily discounted in the lifetime utility function.²⁴ However, note that the estimate of 0.17% is derived for a single contract under a limited contracting space. Multiple financial contracts with contingent parameters would result in more exploitation.²⁵

Policy interventions

Taking the calibrated parameters of the model, this section analyses the impact of three common regulatory policies, similar to those discussed in Chapter 2. Specifically, I consider efficiency and distributional consequences of introducing a ceiling on fees as well as changing the degree of competition and the flexibility of retirement savings.

Ceiling on fees. Suppose that the regulation allows the providing firm to charge only up to a given fraction of the calibrated equilibrium fee, no matter what the offered rate of return. This constraint implies that the firm never offers the rate of

²⁴An alternative efficiency benchmark that is sometimes used in macroeconomic applications would ask which contract maximises the agent's welfare under the assumption that the firm's profits are redistributed back to the agent. For this benchmark, the efficient contract would again be characterised by $R^Z = 5.25\%$. The associated consumption-equivalent loss of consumer welfare is 0.21%. Note that this alternative criterion necessarily coincides with the Pareto criterion only for the case of perfect competition.

²⁵For comparison, Lucas (1987) derives a formula according to which consumption-equivalent losses of consumer welfare associated with the existence of business cycles lie within a range of 0.05% – 0.15%. See also Otrok (2001).

return above the benchmark market rate of 5%. Such an offer would increase the costs of the provider while not generating additional revenue. Thus a ceiling on fees cannot result in a ‘more expensive’ efficient contract with $R^Z = 5.25\%$ being provided by the market. On the contrary, the stricter the ceiling, the lower the equilibrium interest rate. For a given level of competition, the firms retain their profits by offering low-yielding contracts with fees equal to the ceiling. As a result, the entire loss of efficiency is passed onto the agents. Table 3.2.2 presents breakpoints at which gradually lower interest rates are offered in equilibrium.

Table 3.2.2: Ceiling on fees

Fraction of benchmark fee	Market R^Z	Consumer welfare (CE)	Firm’s profits
1.0	5%	-	-
0.95	4.75%	-0.63%	-
0.75	4.50%	-1.47%	-
0.60	4.25%	-2.39%	-
0.50	4%	-3.18%	-1%
0.45	4%	-3.00%	-19%
0.40	4%	-2.83%	-37%

These results are consistent with the theoretical prediction of the simple model from Chapter 2. Naturally, an effective ceiling on fees cannot resolve the issue of ‘inefficiently cheap’ exploitative contracts. Moreover, under fairly competitive conditions when the firms are charging below the agent’s willingness to pay, the intervention does not even achieve redistribution of wealth from the firms to the agents. The firms are able to retain their equilibrium profits by offering progressively lower-yielding contracts and charging the maximum allowed fee. Without a binding ceiling, such contracts would not prevail in equilibrium. A ceiling on fees enforces redistribution only when the firms are already offering the minimum interest rate of 4%.

Regulating competition. The policymaker may control the degree of price

competition observed in the market by affecting the attractiveness of the agent's outside option. This includes removing barriers to entry, explicitly regulating the number of firms, or enabling consumers to switch between providers. Consider an intervention that affects the underlying parameter ξ , changing the markup observable at the benchmark interest rate of 5%. The implications are markedly different from the case of a ceiling on fees. When the firms' markups are cut down, the market provides a more expensive savings contract. Thus a higher degree of price competition improves efficiency as well as redistributes the firm's profits to the agent, see Table 3.2.3.

Table 3.2.3: Regulating competition

Markup	Market R^Z	Consumer welfare (CE)	Firm's profits
0	5.25%	+0.77%	−59%
0.05	5.25%	+0.63%	−44%
0.10	5.25%	+0.49%	−30%
0.15	5.25%	+0.35%	−15%
0.20	5%	-	-
0.25	5%	−0.1%	+10%
0.30	5%	−0.24%	+25%
0.35	5%	−0.39%	+39%
0.40	5%	−0.53%	+54%

While the redistributive consequences of implementing higher degrees of competition are the same as implied by the simple model from Chapter 2, the efficiency consequences are different. Recall that the simple model was solved under the assumption of quasi-linear preferences. When the agent's preferences are linear in the fee, the interest rate offered in market equilibrium is independent of the degree of price competition. That is because the agent's additional willingness to pay for a higher-yielding contract (e.g. $R^Z = 5.25\%$), relative to a lower-yielding contract

(e.g. $R^Z = 5\%$), is independent of the baseline fee level. In contrast, this is no longer true under the extended model, where the agent's preferences are concave in prices. When fees are lower in general, the agent may agree to pay a required premium for a higher-yielding contract.

Liquidity of retirement savings. To proxy for a minimum savings requirement (during the working life) and to consider an additional factor determining the agent's valuation of a savings contract, suppose that the policymaker affects the liquidity of retirement savings by increasing or reducing penalties for early withdrawals. On the one hand, the penalties act as a commitment device against early withdrawals for individuals with self-control problems. On the other hand, flexibility of retirement savings allows to react to (income) shocks in a stochastic setting of a life cycle. A social planner who is able to choose the penalties, would consider this tradeoff between commitment and flexibility and, in a typical case, decide against full flexibility (see Amador et al. 2006). A profit-maximising provider, in contrast, recognises that a completely naïve agent has no taste for commitment, and thus offers fully flexible savings contracts. In other words, private market would underprovide commitment if free to do so. Such contracts, relative to the benchmark case with withdrawal penalties, would increase the agent's valuation and thus exacerbate exploitation. This concern is not limited to a special case of completely naïve agents. Partially naïve individuals would choose insufficient, but costly, commitment, which could also result in more exploitation (Heidhues, Kőszegi 2009).²⁶

Those concerns are reflected in the actual regulatory policies, which seem to dictate the minimum level of commitment. OECD (2016) proposes use of tax incentives to promote saving over long horizons and discourage early withdrawals from private

²⁶In a survey paper, Bryan et al. (2010) discuss other reasons why private markets would not provide sufficient commitment. Importantly, conditions that warrant flexibility may not be observable to the firms and thus contractable upon. See also John (2018) and Schilbach (2018) for empirical evidence on costly under-commitment.

pensions. In the UK, for example, private pension providers have some freedom in determining the withdrawal age, but all withdrawals made before the age of 55 are nonetheless taxed at a rate of 55%. It thus seems reasonable to treat the withdrawal penalty schedule as (exogenously) determined by the policymaker.

Suppose that the shape of the withdrawal penalty schedule over the life cycle is maintained, i.e. the penalties are high during the working life and decrease sharply at retirement, but the level changes as a result of the policy intervention. After the change, the market will provide savings contracts which maximise the consumer's perceived welfare while retaining the firm's profits which are not competed away. Following from the above discussion, the naïve agent's willingness to pay for a contract increases when the savings are made more flexible. That is because naïfs have no preference for commitment. Perhaps counter-intuitively, when the agent is overvaluing the contracts to an even greater extent, he purchases the more expensive (efficient) option and the *actual* consumer welfare improves, see Table 3.2.4.

Table 3.2.4: Liquidity of retirement savings

Withdrawal penalty	Market R^Z	Efficient R^Z	Consumer welfare (CE)	Firm's profits
0.50	5.25%	5.25%	+1.47%	-
0.75	5.25%	5.25%	+0.76%	-
1.0	5%	5.25%	-	-
1.25	5%	5.25%	-0.33%	-

This outcome requires an additional commentary. When the retirement savings are made more flexible, the naïve present-biased agent expects to accumulate more of asset Z , since liquidation is not as costly, but ends up accumulating less than in the benchmark case. This mechanism explains the greater extent of overvaluation of savings contracts with more flexibility. As the agent's willingness to pay increases, he prefers to purchase a more expensive contract, which is now provided by the market.

There is also another reason why an increase in flexibility has a positive impact on the agent's actual welfare. When retirement savings are flexible, the present-biased individual no longer simultaneously saves into an illiquid asset and borrows as much against the liquid asset (credit card), which is detrimental to welfare because of the substantial difference in the interest rates. As a final remark, note that the benchmark penalty schedule was not necessarily socially optimal, which could also contribute to improving the consumer welfare.

Robustness checks

The benchmark calibration is performed under a particular set of modelling assumptions. This section discusses, in turn, the sensitivity to target moments and the impact of the provider's cost function, market structure, and household parametrisation on the calibration. Both qualitative as well as quantitative implications of the model appear robust to these tests. In addition, I demonstrate that the optimal consumption paths are not affected by the simplifying assumption of the negligible impact of fees on the consumption-saving tradeoff. A more detailed discussion and a complete set of numerical results are provided in the appendix.

The benchmark results are not affected by changing the target values and the assumed elasticity of administrative costs γ_1 .

The assumed cost function of the financial provider is indeed a very specific one. I consider two alternative specifications:

$$\text{cost}(t) = k + c(R^Z - R^X)^\gamma$$

and

$$\text{cost}(t) = k + c[(R^Z - R^X)Z(t)]^\gamma$$

The first alternative replaces the administrative cost component which depends on assets under management with a simple fixed cost. The second formulation maintains the assumption of fixed administrative costs, but allows the investment cost to be a function of the agent's total capital earnings (in excess of the 'risk-free' rate R^X). Thus the investment cost is dependent on both the offered rate of return as well as the agent's assets. The calibration strategy and target values are the same as under the benchmark case. Implications of the numerical model regarding the contract design and the impact of contractual choice are unchanged under these alternative specifications.

To test the impact of assuming a perfectly competitive environment, I perform the calibration under a sufficiently low target value for the average markup. The implications of the model are not affected by this modification. Rates of return associated with the market and Pareto-efficient contracts as well as the magnitude of the associated consumer welfare losses are the same as under the benchmark markup of 0.20. Under the assumption of monopoly, in contrast to competitive environments, a savings contract that is offered in equilibrium is simply the one that maximises firm's profits subject to all contract terms (R^Z 's) extracting the entire perceived consumer surplus. For such a calibrated model, an agent would be better off paying a monopolistic fee for a contract with $R^Z = 6\%$ rather than for the profit-maximising contract with $R^Z = 5\%$. The associated consumption-equivalent loss of consumer welfare is much larger at 0.95%. However, in this case, moving from an 'inefficiently cheap' to a 'more expensive' savings contract is *not* a Pareto improvement, because the firm makes strictly higher profits from the exploitative contract.

There is little agreement in the empirical literature on household finance about the 'correct' size of the CRRA parameter θ (e.g. Attanasio, Weber 2010; Laibson

et al. 2017). As a robustness check, LMRT re-estimate their life-cycle model under two arbitrary values of $\theta = 0.5$ and $\theta = 2.0$. The estimated short-run discount factor β is monotonically decreasing in the assumed size of the CRRA parameter θ , while the estimated long-run discount factor δ increases. However, the goodness of fit of the two constrained models is substantially worse than that of the unconstrained model which has $\theta = 1.255$. Similarly, I re-calibrate the supply side of the market for pension products under the two alternative specifications of the household side. When the timing effects are taken into account, the inefficient distortions to contract terms go in the predicted direction. However, the magnitude of the associated losses of consumer welfare is indeed sensitive to the household calibration. Under $\theta = 0.5$, the exploitative contract generates a consumption-equivalent welfare loss of 0.03%, while under $\theta = 2.0$ this estimate equals 0.16%, more in line with the benchmark calibration. For illustration, I additionally show that a monopolist interacting with a household parametrised by $\theta = 0.5$ does offer an ‘inefficiently expensive’ contract, while a monopolist interacting with a household parametrised by $\theta = 2.0$ offers an ‘inefficiently cheap’ contract, consistently with the main prediction of the baseline theoretical model.

Finally, for computational feasibility, the calibrated model assumes that the impact of fees on the agent’s savings behaviour is negligible. I test the validity of this simplifying assumption by re-simulating household behaviour, both actual and expected, given the fee level observed in the benchmark market equilibrium. Allowing for re-optimisation essentially does not affect the consumption and wealth accumulation paths. For instance, while the mean absolute deviation of the expected re-optimised consumption path is just 0.5%, it is only 0.38% for the actual re-optimised consumption path.

3.3 Conclusion

Chapters 2 and 3 study the interaction between a present-biased individual and a private pension provider. By embedding the provider's problem of contract design in a rich life-cycle framework with hyperbolic discounting, in Chapter 3 I attempt to evaluate the quantitative impact of exploitative contracting on efficiency and consumer welfare. Under the benchmark calibration, the savings contract offered in market equilibrium is Pareto-inefficient, i.e. there exists an alternative contract that would increase the agent's utility while retaining the firm's profits. Consistently with the theoretical prediction of the simple model, the equilibrium contract is 'inefficiently cheap'. The prevalence of the exploitative contract terms lowers the agent's wealth at retirement by 10% and generates a consumption-equivalent loss of consumer welfare of 0.17% p.a. This corresponds to \$173 of forgone consumption annually. I argue that this estimated welfare loss arising from exploitative contracting is small, but non-trivial. The calibrated model is subsequently used to examine the quantitative impact of common policy interventions in markets for pension products. I find that interventions which increase the degree of price competition in the market improve efficiency and redistribute wealth from the firm to the agent, while imposing ceilings on charges is detrimental to consumer welfare. What is interesting, while increasing the flexibility of retirement savings exacerbates the overvaluation of all savings contracts by a naïve agent, his willingness to pay increases sufficiently for the agent to select the efficient 'expensive' contract, now provided by the market.

Extending the numerical model to account for a richer contracting space and to allow for dynamic contracting constitute a promising research direction that may be useful in addressing outstanding policy questions.

Appendix

Proposition 2

Solve the problem from the perspective of firm A :

$$\max_{R_A^Z, f_A} \mathcal{P} \times \mathbb{E} \pi_A \quad \text{s.t.}:$$

1. $Z_t = Z_t(\beta, R_A^Z)$
2. $\mathcal{P} = \mathbb{P}(U_0(R_A^Z, f_A, \hat{\beta}) \geq \underline{u})$
3. $\mathbb{E} \pi_A \geq 0$

which can be written more concisely as:

$$\max_{R_A^Z, f_A} \mathbb{P}(U_0(R_A^Z, f_A, \hat{\beta}) \geq \underline{u}) \times \mathbb{E} \pi_A(R_A^Z, f_A, \beta)$$

provided that $\mathbb{E} \pi_A \geq 0$ in solution of the above.

In a first step, fix any $p \in [0, 1]$. Conditional on \underline{u} , for every R_A^Z there is $f_A(R_A^Z)$ such that

$\mathbb{P}(U_0(R_A^Z, f_A(R_A^Z), \hat{\beta}) \geq \underline{u}) = p$. It follows that $f_A(R_A^Z)$ is a function of the agent's belief $\hat{\beta}$. Then, the optimal interest rate R_A^{Z*} maximises:

$$\mathbb{E} \pi_A(R_A^Z, f_A(R_A^Z), \beta)$$

with the following first order condition:

$$\begin{aligned} \frac{\partial \pi_A}{\partial f_A(R_A^Z)} \frac{d f_A(R_A^Z)}{d R_A^Z} &= - \frac{\partial \pi_A}{\partial R_A^Z} \iff \\ \iff \mathbb{E} \sum_{t=1}^T \frac{1}{(1 + R^X)^t} \times \frac{d f_A(R_A^Z)}{d R_A^Z} &= \mathbb{E} \sum_{t=1}^T \frac{1}{(1 + R^X)^t} \frac{d c(R_A^Z, Z_t)}{d R_A^Z} \end{aligned}$$

And similarly for firm B . The second line in the above explicitly distinguishes between the firm's revenue and costs which capture the total impact of R^Z on profits. The above condition has three important implications. First, both firms optimally offer the same interest rate $R_A^{Z*} = R_B^{Z*} = R^{Z*}$. Second, R^{Z*} is independent of ξ . Third, R^{Z*} maximises the *perceived* consumer-firm surplus, as $f_A(R_A^Z)$, which captures the agent's willingness to pay for a contract, is a function of $\hat{\beta}$ and not β . This proves the first statement of Proposition 2.

The optimal choice of $p = \mathbb{P}(U_0(R^{Z*}, f_A, \hat{\beta}) \geq \underline{u})$, or equivalently f_A , maximises the expected profits given the profitability of a single interaction:

$$\max_{f_A} \mathbb{P}(U_0(R^{Z*}, f_A, \hat{\beta}) \geq \underline{u}) \times \mathbb{E} \pi_A(R^{Z*}, f_A, \beta)$$

where

$$\begin{aligned} \mathbb{P}[U_0(R^{Z*}, f_A, \hat{\beta}) \geq \underline{u}] &= \mathbb{P}[U_0(R^{Z*}, f_A, \hat{\beta}) \geq U_0(R^{Z*}, f_B, \hat{\beta})] = \\ &= \mathbb{P}[V_0(\mathbf{C}(\hat{\beta}, R^{Z*}, f_A)) - \xi x \geq V_0(\mathbf{C}(\hat{\beta}, R^{Z*}, f_B)) - \xi(1 - x)] = \\ &= \mathbb{P}[x \leq 0.5 + \frac{V_0(\mathbf{C}(\hat{\beta}, R^{Z*}, f_A)) - V_0(\mathbf{C}(\hat{\beta}, R^{Z*}, f_B))}{2\xi}] = \\ &= 0.5 + \frac{V_0(\mathbf{C}(\hat{\beta}, R^{Z*}, f_A)) - V_0(\mathbf{C}(\hat{\beta}, R^{Z*}, f_B))}{2\xi} \end{aligned}$$

The last line uses the assumption that x is uniformly distributed over $[0, 1]$. The above makes clear that the strength of the motive to lower the price f_A in order to increase the probability of an interaction (or a market share) \mathcal{P} depends on the size of parameter ξ . The maximisation of expected profits by firm A yields the following first order condition:

$$\begin{aligned}
-\frac{d\mathbb{P}[U_0(R^{Z*}, f_A, \hat{\beta}) \geq u]}{df_A} &= \frac{d\mathbb{E}\pi_A(R^{Z*}, f_A, \beta)}{df_A} \times \frac{\mathbb{P}[U_0(R^{Z*}, f_A, \hat{\beta}) \geq u]}{\mathbb{E}\pi_A(R^{Z*}, f_A, \beta)} \iff \\
\iff -\frac{dV_0(\mathbf{C}(\hat{\beta}, R^{Z*}, f_A))}{df_A} \times \frac{1}{2\xi} &= \frac{d\mathbb{E}\pi_A(R^{Z*}, f_A, \beta)}{df_A} \times \frac{\mathbb{P}[U_0(R^{Z*}, f_A, \hat{\beta}) \geq u]}{\mathbb{E}\pi_A(R^{Z*}, f_A, \beta)}
\end{aligned}$$

The above condition implies that the equilibrium fee f_A^* is monotonically increasing in ξ , ranging from competitive pricing to monopolistic pricing. The competitive price is determined by the non-negativity constraint $\mathbb{E} \pi_A \geq 0$, while the monopolistic price is a function of the agent's outside option v . This formula highlights the trade-off between profits from a single interaction and a market share. Notice that parameter ξ determines sensitivity of firm's A market share to its price. For low values of ξ , firm A has more incentives to post a low fee and substantially increase its market share, while this motive is not as strong for higher values of ξ .

Finally, by symmetry, both providers offer the same equilibrium fee $f_A^* = f_B^* = f^*$ and the indifferent consumer is located at $x = 0.5$. This proves the second statement of Proposition 2.

Benchmark targets

As a benchmark case, the numerical model is calibrated to match the cost-to-assets ratio of 0.005, the share of administrative costs of 0.50, the markup of 0.20, and the equilibrium interest rate of 5%. All numerical values are calculated at the means for computational convenience. Parameter γ_1 , the elasticity of administrative costs, is set equal to 0.5. This section discusses the choice these values in turn.

Operational costs expressed as a fraction of assets under management (the cost-to-assets ratio) is a usual metric used by the empirical literature on expenses of pension funds. The observed cost-to-assets ratio tends to vary considerably across

firms and countries, but typical values lie within a range of [0.1%, 1.2%], with US pension funds reporting costs towards a lower end of the scale (Bateman, Mitchell 2004; Bauer et al. 2010; Bikker, de Dreu 2009; Bikker et al. 2012; OECD 2017).

Only the analysis of Bikker and de Dreu (2009) allows to directly compare the relative importance of administrative and investment costs in determining the total costs of pension funds. On average, administrative costs are 50% greater than investment costs, but there is much more heterogeneity in the distribution of administrative costs. Moreover, investment costs tend to be under-reported. In comparison, Bikker et al. (2012) remark only that administrative costs constitute “a very large proportion” of the total cost.

There is little reliable data on markups of pension funds, although a few papers stress the need to distinguish between costs and charges in the pension industry due to imperfect price competition (e.g. Bikker et al. 2012; OECD 2017). According to the report by the Financial Conduct Authority (2017), an average profit margin in asset management industry in the UK was as high as 36%. I interpret this as an upper bound for markups in the pension industry.

Typical annual rates of return of pension funds lie within a range of from 3% to 7% in real terms (Bauer et al. 2010; OECD 2017), although there is considerable variation across time and across countries. In addition, setting $R^{Z*} = 5\%$ as a benchmark target reflects the return on the illiquid asset assumed by Laibson et al. (2017). This is an important point of consideration, as the estimates of the household preference parameters strongly depend on a calibration of the economic environment. As two extreme alternatives, Laibson et al. consider rates of return of 3.38% and 6.59%.

Bateman and Mitchell (2004) find that the elasticity of administrative costs of Australian pension funds with respect to assets under management is 0.46, and 0.87

for a subset of retail funds. Bikker et al. (2012), who use a less inclusive definition of administrative costs, estimate the elasticity of 0.19 for a sample of Australian, Canadian, Dutch and American pension funds. For a subset of the US pension funds, the estimate equals 0.23.

Sensitivity analysis

Table 3.3.1 presents the sensitivity of the numerical results to changes in the target values and the assumed elasticity of administrative costs γ_1 .

Table 3.3.1: Sensitivity analysis

Target	Calibrated parameters	Pareto- efficient R^Z	CE wel- fare loss	Retirement savings
Benchmark	$c_1 = 1.3797$ $\gamma_1 = 0.5$ $c_2 = 2.1918 \times 10^{12}$ $\gamma_2 = 5.75$ $\xi = 0.0808$	5.25%	0.17%	-10%

$\gamma_1 = 0.35$	$c_1 = 8.9469$ $\gamma_1 = 0.35$ $c_2 = 2.6523 \times 10^{12}$ $\gamma_2 = 5.80$ $\xi = 0.0808$	5.25%	0.17%	-10%
$\gamma_1 = 0.65$	$c_1 = 0.2100$ $\gamma_1 = 0.65$ $c_2 = 1.8113 \times 10^{12}$ $\gamma_2 = 5.70$ $\xi = 0.1010$	5.25%	0.17%	-10%
Cost-to- assets ratio 0.0035	$c_1 = 0.9658$ $\gamma_1 = 0.5$ $c_2 = 1.4697 \times 10^{15}$ $\gamma_2 = 7.55$ $\xi = 0.0517$	5.25%	0.17%	-10%
Cost-to- assets ratio 0.0065	$c_1 = 1.7936$ $\gamma_1 = 0.5$ $c_2 = 3.5482 \times 10^{10}$ $\gamma_2 = 4.60$ $\xi = 0.1010$	5.25%	0.17%	-10%

Admin share 0.40	$c_1 = 1.1038$ $\gamma_1 = 0.5$ $c_2 = 1.8221 \times 10^{11}$ $\gamma_2 = 5.05$ $\xi = 0.0808$	5.25%	0.17%	-10%
Admin share 0.60	$c_1 = 1.6557$ $\gamma_1 = 0.5$ $c_2 = 6.5671 \times 10^{13}$ $\gamma_2 = 6.70$ $\xi = 0.1010$	5.25%	0.17%	-10%
Markup 0.15	$c_1 = 1.3797$ $\gamma_1 = 0.5$ $c_2 = 2.6523 \times 10^{12}$ $\gamma_2 = 5.80$ $\xi = 0.0646$	5.25%	0.16%	-10%
Markup 0.25	$c_1 = 1.3797$ $\gamma_1 = 0.5$ $c_2 = 1.8113 \times 10^{12}$ $\gamma_2 = 5.70$ $\xi = 0.1010$	5.25%	0.18%	-10%

$R^{Z*} = 4.75\%$	$c_1 = 1.2760$ $\gamma_1 = 0.5$ $c_2 = 6.7542 \times 10^{12}$ $\gamma_2 = 5.90$ $\xi = 0.0646$	5%	0.07%	-12%
$R^{Z*} = 5.25\%$	$c_1 = 1.4717$ $\gamma_1 = 0.5$ $c_2 = 2.8226 \times 10^{12}$ $\gamma_2 = 5.95$ $\xi = 0.1010$	5.50%	0.16%	-8%

The sensitivity analysis shows that the quantitative implications of the model are robust to distorting the target values. Notice that the above also makes apparent the underlying mechanics of the numerical model. First, equilibria characterised by higher markups require greater monopolistic power of the provider and thus the calibrated ξ takes greater values. Second, recall that the cost conditions jointly determine parameters c_2 and γ_2 and that is why higher values of c_2 are associated with higher values of γ_2 . Third, the total cost function needs to be ‘steep enough’ (convex) in order to generate an interior R^{Z*} . That is why calibrations that assume higher γ_1 require lower γ_2 and calibrations with a greater share of administrative costs require higher γ_2 .

Robustness checks

Table 3.3.2 presents the numerical results for a battery of robustness checks.

Table 3.3.2: Robustness checks

Check	Calibrated parameters	Pareto- efficient R^Z	CE wel- fare loss	Retirement savings
Benchmark	$c_1 = 1.3797$ $\gamma_1 = 0.5$ $c_2 = 2.1918 \times 10^{12}$ $\gamma_2 = 5.75$ $\xi = 0.0808$	5.25%	0.17%	-10%
cost = $k + c(R^Z - R^X)^\gamma$	$k = 656.31$ $c = 4.6996 \times 10^{12}$ $\gamma = 5.95$ $\xi = 0.0517$	5.25%	0.17%	-10%
cost = $k + c[(R^Z - R^X)Z(t)]^\gamma$	$k = 656.31$ $c = 4.6454 \times 10^{-17}$ $\gamma = 4.90$ $\xi = 0.1262$	5.25%	0.17%	-10%

Competition (markup of −0.13)	$c_1 = 1.3797$ $\gamma_1 = 0.5$ $c_2 = 2.1918 \times 10^{12}$ $\gamma_2 = 5.75$ $\xi = 0.0001$	5.25%	0.18%	−10%
Monopoly	$c_1 = 1.3797$ $\gamma_1 = 0.5$ $c_2 = 6.9809 \times 10^{11}$ $\gamma_2 = 5.45$ $\xi = 0.7482$	6%	0.95%	−33%
$\theta = 0.5$ $(\beta = 0.7493,$ $\delta = 0.9725)$	$c_1 = 0.9717$ $\gamma_1 = 0.5$ $c_2 = 1.0985 \times 10^{17}$ $\gamma_2 = 8.80$ $\xi = 21.3821$	5.25%	0.03%	−80%
$\theta = 0.5$ $(\beta = 0.7493,$ $\delta = 0.9725);$ monopoly	$c_1 = 0.9717$ $\gamma_1 = 0.5$ $c_2 = 5.1233 \times 10^{16}$ $\gamma_2 = 8.60$ $\xi = 327.6247$	4%	1.04%	+98%

$\theta = 2.0$ ($\beta = 0.4371$, $\delta = 0.9919$	$c_1 = 1.5718$ $\gamma_1 = 0.5$ $c_2 = 6.0258 \times 10^9$ $\gamma_2 = 4.15$ $\xi = 0.0001$	5.25%	0.16%	-5%
$\theta = 2.0$ ($\beta = 0.4371$, $\delta = 0.9919$; monopoly	$c_1 = 1.5718$ $\gamma_1 = 0.5$ $c_2 = 1.0831 \times 10^9$ $\gamma_2 = 3.70$ $\xi = 0.0008$	6%	0.91%	-15%

These results require additional explanation. First, robustness checks regarding functional form of the firm's cost function are performed under the benchmark target values. Second, robustness checks imposing an arbitrary degree of competition use the same target values, save for the provider's markup. The case of perfect competition targets an average markup of -0.13 , under which the discounted expected value of the firm's profits is non-negative. It becomes negative for an average markup of -0.14 . The case of monopoly has a provider offering a savings contract that maximises profits subject to fees associated with different rates of return extracting the entire perceived consumer surplus. An agent would be better off paying a monopolistic fee for a more expensive savings contract with $R^Z = 6\%$, however the movement from $R^Z = 5\%$ can no longer be interpreted as a Pareto improvement. That is because the firm strictly prefers to offer an inefficiently cheap savings contracts.

An alternative parametrisation of the household with $\theta = 0.5$ again produces an

inefficiently cheap contract in market equilibrium. That is because in a dynamic setting the timing effects can dominate the standard relationship between the interest rate and optimal savings, conditional on the size of the CRRA parameter. For this parametrisation and for the rates of return exceeding 4.5%, the household starts saving for retirement earlier on in the life cycle when the interest rate increases and thus the total amounts saved are *decreasing* in R^Z despite $\theta < 1$. As a result, the firm increases its profit at the margin by offering an inefficiently cheap savings contract. In contrast, a monopolist interacting with the household parametrised by $\theta = 0.5$ offers an ‘inefficiently expensive’ contract, as predicted by the stylised model. Because of a high intertemporal elasticity of substitution, the contract terms have a huge impact on the agent’s savings. On the other hand, a parametrisation with $\theta = 2.0$ produces inefficiently cheap savings contracts in market equilibria, both under a competitive environment and under a monopoly.

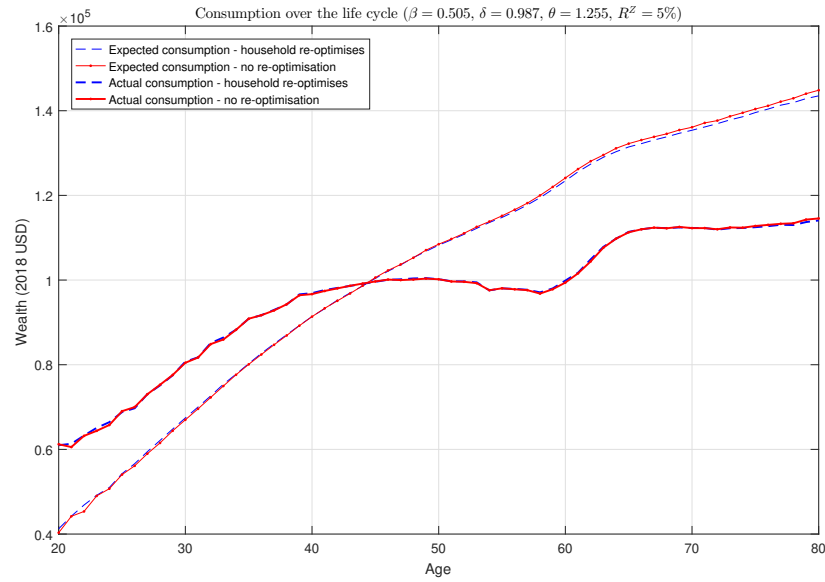


Figure 3.3.1: Consumption paths (expected and actual)
Source: Author’s calculations

For computational feasibility, the benchmark numerical model assumes that the agent's saving decisions are independent of fees charged by the provider. I check that this is not an overly restrictive assumption by simulating behaviour of a household that re-optimises their consumption and wealth accumulation paths given the fees charged in equilibrium of the benchmark model. As Figure 3.3.1 above demonstrates, the resulting consumption paths do not deviate significantly from the benchmark, constrained-optimised paths. The mean absolute deviation of the actual re-optimised consumption path is only 0.38%, while for the expected consumption paths the mean absolute difference is 0.5%. The re-optimised model predicts a slightly higher actual accumulation of asset X and a slightly lower accumulation of asset Z . However, the difference in holdings of the illiquid wealth at retirement is not large at 2.8%.

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